# Study of WT, KLT and Adaptive Filter

**Saurabh Rai**[#1], **Anand Vardhan Bhalla**[#2]
[#1]*M.Tech scholar, Babulal Tarabai Institute of Research and Technology, Sironja, Sagar,Madhya Pradesh, India.*
[#2]*Assistant Professor, Electronics and Communication Engineering Department, Babulal Tarabai Institute of Research and Technology, Sironja, Sagar,Madhya Pradesh, India.*
[1]*saurabhgoodboy@gmail.com*
[2]*anandvardhanbhalla@gmail.com*

***Abstract*— Visual information transmitted in the form of digital images is becoming a major method of communication in the modern age, but the image obtained after transmission is often corrupted with noise. The received image needs processing before it can be used in applications. Image denoising involves the manipulation of the image data to produce a visually high quality image. Different noise models including additive and multiplicative types are prevalent. They include Gaussian noise, salt and pepper noise, speckle noise and Brownian noise. In the past few decades, many noise reduction techniques have been developed for removing noise and retaining edge details in images. Hence, it is necessary to have knowledge about the noise present in the image so as to select the appropriate denoising algorithm. The filtering approach has been proved to be the best when the image is corrupted with Gaussian noise. The WT (Wavelet transform) and KLT (Karhunen-Loeve transform) based approach finds applications in denoising images corrupted with Gaussian noise. This paper contains a review of WT (Wavelet transform), KLT (Karhunen-Loeve transform) and adaptive filter as they are the popularly used for denoising an image affected by white Gaussian noise.**

***Keywords*— WT (Wavelet transform), KLT (Karhunen-Loeve transform) , Adaptive filter, Image processing, Noise.**

## I. Introduction

It is well known to scientist and engineer that signals do not exist without noise, which may be negligible under certain conditions. . Digital images play an important role both in daily life applications such as satellite television, magnetic resonance imaging, computer tomography as well as in areas of research and technology such as geographical information systems and astronomy. Data sets collected by image sensors are generally contaminated by noise. Imperfect instruments, problems with the data acquisition process, and interfering natural phenomena can all degrade the data of interest. Furthermore, noise can be introduced by transmission errors and compression. However, there are many cases in which the noise corrupts the signals in a significant manner, and it must be removed from the data in order to proceed with further data analysis. The process of noise removal from an image is generally referred to as image denoising or simply denoising. It can be seen that the noise adds high-frequency components to the original signal which is smooth. This is a characteristic of noise. Many dots can be spotted in a Photograph taken with a digital camera under low lighting conditions. Appearance of dots is due to the real signals getting corrupted by noise. Although the term "image denoising" is general, it is usually devoted to the recovery of a digital signal that has been contaminated by additive white Gaussian noise (AWGN), rather than other types of noise. The optimization criterion according to which the performance of a denoising algorithm is measured is usually taken to be peak signal to noise ratio (PSNR) and mean squared error (MSE)-based, between the original image and its reconstructed version. This common criterion is used mostly due its computational simplicity. Moreover, it usually leads to expressions which can be dealt with analytically. However, this criterion may be inappropriate for some tasks in which the criterion is perceptual quality driven, though perceptual quality assessment itself is a difficult problem, especially in the absence of the original signal.

There is a wide range of applications in which denoising is important. Examples are medical image/signal analysis, data mining, radio astronomy and there are many more. Each application has its special requirements. For example, noise removal in medical signals requires specific care, since denoising which involves smoothing of the noisy may cause the loss of fine details.

Many image-processing algorithms such as pattern recognition need a clean image to work effectively. Random and uncorrelated noise samples are not compressible. Such concerns underline the importance of denoising in image and video processing. Thus, denoising is often a necessary and the first step to be taken before the images data is analyzed. It is necessary to apply an efficient denoising technique to compensate for such data corruption. Image denoising still remains a challenge for researchers because noise removal introduces artifacts and causes blurring of the images.

## II. Image Representation

A 2-dimensional digital image can be represented as a 2-dimensional array of data $s(x,y)$, where $(x,y)$ represent the pixel location. The pixel value corresponds to the brightness of the image at location $(x,y)$. Some of the most frequently used image types are binary, gray-scale and colour images.

Binary images are the simplest type of images and can take only two discrete values, black and white. Black is represented with the value ‘0’ while white with 1’. Note that a binary image is generally created from a gray-scale image. A binary image finds applications in computer vision areas where the general shape or outline information of the image is needed. They are also referred to as 1 bit/pixel images.

Gray-scale images are known as monochrome or one-colour images. They contain no colour information. They represent the brightness of the image. This image contains 8 bits/pixel data, which means it can have up to 256 (0-255) different brightness levels. A 0'represents black and '255' denotes white. In between values from 1 to 254 represent the different gray levels. As they contain the intensity information, they are also referred to as intensity images.

Colour images are considered as three band monochrome images, where each band is of a different colour. Each band provides the brightness information of the corresponding spectral band. Typical colour images are red, green and blue images and are also referred to as RGB images. This is a 24 bits/pixel image.

## III. ARTIFACTS OF DENOISING

Mostly denoising adds its own noise to an image. Some of the artifacts created by denoising are as follows:

1. ***Blur***: Attenuation of high spatial frequencies may result in smooth edges in the image.
2. ***Ringing/Gibbs Phenomenon:*** Truncation of high frequency transform coefficients may lead to oscillations along the edges or ringing distortions in the image.
3. ***Staircase Effect:*** Aliasing of high frequency components may lead to stair-like structures in the image.
4. ***Checkerboard Effect:*** Denoised images may sometimes carry checkerboard structures.
5. ***Wavelet Outliers:*** These are distinct repeated wavelet-like structures visible in the denoised image and occur in algorithms that work in the wavelet domain.

## IV. WAVELET TRANSFORM (WT)

The origins of the wavelet analysis can be traced to the 1909 Haar and various "atomic decompositions" in the history of mathematics. The current use of the name "wavelet" is due to Grosman's and Morlet's work on geophysical signal processing, which led to the formalization of the continuous wavelet transform. In the development of wavelets, the ideas from many different fields (including sub band coding and computer vision) have merged.

Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. It has an oscillating wavelike characteristic & it as time-scale and time-frequency analysis tools have been widely used in reconstruction and still growing. In wavelet analysis, the scale that used to look at data plays an important role. Wavelet algorithms process data at different scales or resolutions. Looking at a signal with a large "window," that would notice global features. Similarly, looking at a signal with a small "window," that would notice localized features.

The multiresolution possibility of wavelet transform (WT) makes it a powerful tool for signal processing. Applications in which frequency response varies in time, wavelet transform is proved to be more useful. The wavelet coefficients are computed as a convolution of the signal and the scaled wavelet function, which can be interpreted as a dilated band pass filter because of its band pass like spectrum. These coefficients give a measure of similarity between the frequency content of a signal and that of a chosen wavelet function. The discrete wavelet transform (DWT) needs less space as it utilizes the space saving coding which is based on the fact that wavelet families are orthogonal or biorthogonal and hence does not require redundant analysis. The discrete wavelet transform corresponds to the sampling of its continuous version using a dyadic grid, which means that the scales and translations are power of two. Thresholding is a non-linear technique, which operates on one wavelet coefficient at a time. In this each coefficient is thresholded by comparing against threshold. If the coefficient is smaller in comparison to threshold then the coefficient is set to zero; otherwise it is kept or modified to some other level.

The wavelet family is generated from a unique prototype function that is called a mother wavelet. Given a real variable x, the function $\psi(x)$ is called a mother wavelet provided that it oscillates, averaging to zero and that is well localized (i.e., rapidly decreases to zero when |x| tends to infinity). By convention it is centered around x = 0, and has a unit norm $||\psi(x)||$. In practice, applications impose additional requirements among which, a given number of vanishing moments $N_v$.

$$\int_{-\infty}^{\infty} x^k \psi(x) dx = 0, \quad 0 < k \leq N_v - 1$$

The mother wavelet $\psi(x)$, generates the other wavelets $\psi_{a,b}(x)$, a >0, b ε R, of the family by change of scale a (i.e., by dilation) and by change of position b (i.e., by translation),

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi(\frac{x-b}{a}), \quad a > 0, \ b \in \mathbb{R}$$

## V. KARHUNEN-LOEVE TRANSFORM (KLT)

KLT is known for its de-correlation, energy concentration and under measuring of the MSE. KLT is built on statistical-based properties. The WT based on waveform transform. They are based on different foundation. The outstanding advantage of KLT is a good de-correlation.

The KLT is a mathematical tool superior to the FFT as its accuracy applies to any finite bandwidth, rather than applying to infinitely small bandwidth only (i.e. to monochromatic signals) as the FFT does. Also, the KLT applies to both stationary and non-stationary processes, but the FFT works only for stationary input stochastic processes. The KLT is defined for any finite time interval, but the FFT is plagued by the "window" problems. For the KLT it needs high computational burden because of no "fast" KLT. Comparing with FFT, KLT is fast algorithm.

### *A.Calculation of KLT*

#### *1) Estimation of Covariance:*

The Karhunen-Loeve Transform (KLT) is calculated by finding the eigenvectors of the covariance matrix, which requires an estimate of the covariance matrix. If the entire signal is available the covariance matrix can be estimated from n data samples as

$$[\widehat{\mathbf{C}}]_x = \frac{1}{n}\sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^T ,$$

where $\mathbf{x}_i$ is a sample data vector. If only portions of the signal are available then care must be taken to ensure that the estimate is representative of the entire signal. In the extreme, if only one data vector is used then only one nonzero eigenvalue exists, and its eigenvector is simply the scaled version of the data vector. For typical images, it is rarely the case that their covariance matrix has any zero eigenvalues.

#### *2) Calculation of Eigenvectors:*

A simple approach is the Jacobi method. It develops a sequence of rotation matrices, [**P**]i that diagonalizes [**C**] as

$$[\mathbf{D}] = [\mathbf{V}]^T[\mathbf{C}][\mathbf{V}] ,$$

where [**D**] is the desired diagonal matrix and [**V**] = $[\mathbf{P}]_1[\mathbf{P}]_2[\mathbf{P}]_3 \cdot \cdot \cdot$ . Each $[\mathbf{P}]_i$ rotate in one plane to remove one of the off-diagonal elements. It is an iterative technique which is terminated when the off-diagonal values are close to zero within some tolerance. Upon termination, the matrix [**D**] contains the eigenvalues on the diagonals and the columns of [**V**] are the basis vectors of the KLT. While this technique is quite simple, for larger matrices it can take a large number of calculations for convergence. A more efficient approach for larger, symmetric matrices divides the problem into two stages. The Householder algorithm can be applied to reduce a symmetric matrix into a tridiagonal form in a finite number of steps. Once the matrix is in this simpler form, an iterative method such as QL factorization can be used to generate the eigenvalues and eigenvectors. The advantage of this approach is that the factorization on the simplified tridiagonal matrix typically requires less iteration than the Jacobi method. Recently, there has been some interest in iterative methods of principal components extraction that do not require the calculation of a covariance matrix. These techniques update the estimate of the eigenvectors for each input training vector.

## VI. LINEAR FILTERING APPROACH

Filters play a major role in the image restoration process. The basic concept behind image restoration using linear filters is digital convolution and moving window principle. Let w(x) is the input signal subjected to filtering, and z(x) is the filtered output. If the filter satisfies certain conditions such as linearity and shift invariance, then the output filter can be expressed mathematically in simple form as

$$z(x) = \int w(t)h(x-t)dt$$

where h(t) is called the point spread function or impulse response and is a function that completely characterizes the filter. The integral represents a convolution integral and, in short, can be expressed as z = w* h.
For a discrete case, the integral turns into a summation as

$$z(i) = \sum_{-\infty}^{+\infty} w(t)h(i-t)$$

Although the limits on the summation in are ∞, the function h(t) is usually zero outside some range. If the range over which h(t) is non-zero is (-k, +k), then the above can be written as

$$z(i) = \sum_{i-k}^{i+k} w(t)h(i-t)$$

This means that the output z(i) at point i is given by a weighted sum of input pixels surrounding i where the weights are given by h(t). To create the output at the next pixel i+1, the function h(t) is shifted by one and the weighted sum is recomputed. The total output is created by a series of shift-multiply-sum operations, and this forms a discrete convolution. For the 2-dimensional case, h(t) is h(t,u), and equation becomes

$$z(i,j) = \sum_{t=i-k}^{i+k} \sum_{u=j-l}^{j+l} w(t,u)h(i-t,j-u)$$

Values of h(t,u) are referred to as the filter weights, the filter kernel, or filter mask. For reasons of symmetry h(t,u) is always chosen to be of size m×n where m and n are both odd (often m=n). In physical systems, the kernel h(t,u) must always be non-negative which results in some blurring or averaging of the image. If the coefficients are alternating positive and negative, the mask is a filter that returns edge information only. The narrower the h(t,u), the better the system in the sense of less blurring. In digital image processing, h(t,u) maybe defined arbitrarily and this gives rise to many types of filters. The weights of h(t,u) may be varied over the image and the size and shape of the window can also be varied. These operations are no longer linear and no longer convolutions. They become moving window operations. With this flexibility, a wide range of linear, non-linear and adaptive filters may be implemented.

An adaptive filter does a better job of denoising images compared to the averaging filter. The fundamental difference between the mean filter and the adaptive filter lies in the fact that the weight matrix varies after each iteration in the adaptive filter while it remains constant throughout the iterations in the mean filter. Adaptive filters are capable of denoising non-stationary images, that is, images that have abrupt changes in intensity. Such filters are known for their ability in automatically tracking an unknown circumstance. In general, an adaptive filter iteratively adjusts its parameters during scanning the image to match the image generating mechanism. This mechanism is more significant in practical images, which tend to be non-stationary.
The adaptive filter is known for its simplicity in computation and implementation. The basic model is a linear combination of a stationary low-pass image and a non-stationary high-pass component through a weighting function. Thus, the function provides a compromise between resolution of genuine features and suppression of noise.

## VII. CONCLUSION

Both the WT and KLT are used for image denoising. KLT is not as popular as WT, the reason caused by its different mathematical structure. Adaptive filter is a linear filter which is also used to filter the noise present in the image. All the three WT, KLT and adaptive filter shows excellent performance when the image is affected by the white Gaussian noise.

## REFERENCES

[1] M.Neelima1, Md. Mahaboob Pasha , "*Wavelet Transform Based On Image Denoising Using Thresholding Techniques",* International Journal of Advanced Research in Computer and Communication Engineering Vol. 3, Issue 9, September 2014.

[2] F Jin, P Fieguth, L. Winger and E. Jernigan, *"Adaptive Wiener Filtering Of Noisy Images And Image Sequences",* Department of Systems Design Engineering University of Waterloo Waterloo, Ontario, Canada, N2L 3G1.

[3] Miss. Pallavi Charde, **"** *A Review On Image Denoising Using Wavelet Transform And Median Filter Over AWGN Channel",* International Journal Of Technology Enhancements And Emerging Engineering Research, Vol 1, Issue 2013 IJTEEE.

[4] Kanwaljot Singh Sidhu1, Baljeet Singh Khaira2, Ishpreet Singh Virk3, " *Medical Image Denoising In The Wavelet Domain Using Haar And DB3 Filtering*", International Refereed Journal of Engineering and Science (IRJES) ISSN (Online) 2319-183X, (Print) *2319-1821* Volume 1, Issue 1 (September 2012), PP.001-008.

[5] Anutam and Rajni, *"Performance Analysis Of Image Denoising With Wavelet Thresholding Methods For Different Levels of Decomposition",* The International Journal of Multimedia & Its Applications (IJMA) Vol.6, No.3, June 2014.

[6] R.M.Suresh, *"Analysis of Image Denoising using Wavelet Coefficient and Adaptive Subband Thresholding Technique",* IJCSI International Journal of Computer Science Issues, Vol. 8, Issue 6, No 1, November 2011,ISSN: 1694 0814.

[7] Jyotsna Patil , Sunita Jadhav, *"A Comparative Study of Image Denoising Technique",* International Journal of Innovative Research in Science, Engineering and Technology Vol. 2, Issue 3, March 2013.

[8] Akanksha Deo DBV Singh, *" Denoising of ECG Signals with Adaptive Filtering Algorithms & Patch Based Method",* International Journal of Computer Networks and Wireless Communications (IJCNWC), ISSN: 2250-3501 Vol.3, No3, June 2013

[9] S.Kalavathy and R.M.Suresh, *"Analysis of Image Denoising using Wavelet Coefficient and Adaptive Subband Thresholding Technique"* , IJCSI International Journal of Computer Science Issues, Vol. 8, Issue 6, No 1, November 2011 ISSN (Online): 1694-0814.

[10] Sinan Gezici , Ismail Yilmaz , Omer N. Gerek_ ,A. Enis Cetin, "*Image Denoising Using Adaptive Subband Decomposition",* ©2001 IEEE.

[11] Namrata Dewanga, Agam Das Goswam*, "Image Denoising Using Wavelet Thresholding Methods",* International Journal of Engineering Sciences & Management. Int. J. of Engg. Sci. & Mgmt. (IJESM), Vol. 2, Issue 2: April-June: 2012, 271 -275.

[12] Miss. Pallavi Charde, *" A Review On Image Denoising Using Wavelet Transform And Median Filter Over AWGN Channel",* International Journal Of Technology Enhancements And Emerging Engineering Research, Vol 1, Issue 4 44 ISSN 2347-4289.

[13] Iram Sami, Abhishek Thakur, Rajesh Kumar, *"Image Denoising for Gaussian Noise Reduction in Bionics Using DWT Technique",* IJECT Vol. 4, Issue April - June 2013.

[14] Akhilesh Bijalwan, Aditya Goyal, Nidhi Sethi, *"Wavelet Transform Based Image Denoise Using Threshold Approaches",* International Journal of Engineering and Advanced Technology (IJEAT) ISSN: 2249 – 8958, Volume-1, Issue-5, June 2012.

[15] Chandrika Saxena , Prof. Deepak Kourav, *"Noises and Image Denoising Techniques: A Brief Survey",* International Journal of Emerging Technology and Advanced Engineering, ISSN 2250-2459, ISO 9001:2008 Certified Journal, Volume 4, Issue 3, March 2014.

[16] J. N. Ellinas, T. Mandadelis, A. Tzortzis, L. Aslanoglou, *"Image de-noising using wavelets".*

[17] Daoqiang Zhang, Songcan Chen, *"Fast image compression using matrix K-L transform",* Department of Computer Science and Engineering, Nanjing University of Aeronautics & Astronautics, Nanjing 210016, P.R. China.

[18] Sachin D Ruikar,Dharmpal D Doye, *"Wavelet Based Image Denoising Technique",* International Journal of Advanced Computer Science and Applications (IJACSA), Vol. 2, No.3, March 2011.

[19] Mustafa U. Torun and Ali N. Akansu, *"An Efficient Method to Derive Explicit KLT Kernel for First-Order Autoregressive Discrete Process",* New Jersey Institute of Technology, Department of Electrical & Computer Engineering, University Heights, Newark, NJ 07102 USA.

 

[20] Raghuram Rangarajan Ramji Venkataramanan Siddharth Shah, *"Image Denoising Using Wavelets",* December 16, 2002.

[21] Aleksandra Pizurica, *"Image Denoising Using Wavelets and Spatial Context Modeling",* Vakgroep Telecommunicatie en Informatieverwerking Voorzitter: Prof. dr. ir. H. Bruneel Academiejaar 2001-2002.

[22] Rajat Singh, D.S. Meena, *"Image Denoising Using Curvelet Transform*" Department of Computer Science and Engineering National Institute of Technology, Rourkela.

[23] Adrian E. Villanueva- Luna1, Alberto Jaramillo-Nuñez1, Daniel Sanchez-Lucero1, Carlos M. Ortiz-Lima1, J. Gabriel Aguilar-Soto1, Aaron Flores-Gil2 and Manuel May-Alarcon2, *" De-Noising Audio Signals Using MATLAB Wavelets",* Instituto Nacional de Astrofisica, Optica y Electronica (INAOE) ,Universidad Autonoma del Carmen (UNACAR)Mexico.

[24] Stephen Wolf, Margaret Pinson, *"Algorithm for Computing Peak Signal to Noise Ratio (PSNR) of a Video Sequence with a Constant Delay",* Geneva, February 2-6, 2009.

[25] Adrian E. Villanueva- Luna1, Alberto Jaramillo-Nuñez1, Daniel Sanchez-Lucero1, Carlos M. Ortiz-Lima1, J. Gabriel Aguilar-Soto1, Aaron Flores-Gil2 and Manuel May-Alarcon2, *"De-Noising Audio Signals Using MATLAB Wavelets"* Instituto Nacional de Astrofisica, Optica y Electronica (INAOE) ,Universidad Autonoma del Carmen (UNACAR) Mexico.

[26] Donoho, D.L., I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," Biometrika, Vol. 81, pp. 425–455.

[27] Claudio Maccone, "*Advantages of Karhunen-Loeve transform over fast Fourier transform for planetary radar and space debris detection*", International academy of Astronautics, Via martorelli 43, Torino(TO) 10155, Italy. Available online 27 October 2006.

[28] Ryan S. Overbeck, *"Adative Wavelet rendering",* Donnery Columbia University. Z. Ravi *Rama moorthi , University of California, Berkeley.*

[29] K.Ramachandran, S.LoPresto and M.Orchard, *"Image coding based on mixture modeling of wavelet coefficients and a fast estimation quantization framework"* in Proc. Data compression Conf., Snowbird, UT, March 1997.

[30] C Sidney Burrus, Ramesh A Gopinath, and Haitao Guo, "*Introduction to wavelet and wavelet transforms*", Prentice Hall1997.S. Mallat, A Wavelet Tour of Signal Processing, Academic, New York, second edition, 1999.

[31] Raghuveer M. Rao., A.S. Bopardikar "*Wavelet Transforms: Introduction To Theory And Application"* Published By Addison-Wesley 2001 pp1-126.

[32] Jaideva Goswami Andrew K. Chan, "*Fundamentals Of Wavelets Theory, Algorithms, And Applications",* John Wiley Sons

[33] H. A. Chipman, E. D. Kolaczyk, and R. E. McCulloch: "*Adaptive Bayesian wavelet shrinkage"*, J. Amer. Stat. Assoc., Vol. 92, No 440, Dec. 1997, pp. 1413-1421

[34] Sasikala, P. (2010). "*Robust R Peak and QRS detection in Electrocardiogram using Wavelet Transform*", International Journal of Advanced Computer Science and Applications - IJACSA, 1(6), 48-53.

[35] Kekre, H. B. (2011). Sectorization of Full Kekre "Wavelet *Transform for Feature extraction of Color Images*". International Journal of Advanced Computer Science and Applications - IJACSA, 2(2), 69-74.

[36] Suresh Kumar, Papendra Kumar, Manoj Gupta, Ashok Kumar Nagawat, *"Performance Comparison of Median and Wiener Filter in Image De-noising"* International Journal of Computer Applications (0975 – 8887) Volume 12– No.4, November 2010.

[37] I. Daubechies and W.Swwldens, *"Factoring wavelet transforms into lifting steps",*J.Fourier Anal. Appl.. Vol 4, no 3, PP-245-267 1998.

[38] R.Calderbank, I. Daubechies, W. Sweldens and B.L.Yeo, *"Wavelet transform that map integers to integers"* Appl Comput,Harmon Anal.,vol 5, No 3, pp332- 369,1998.

[39] Gao Zhing, Yu Xiaohai, "*Theory and application of MATLAB Wavelet analysis tools"*, National defense industry publisher,Beijing,pp.108-116, 2004.

[40] Aglika Gyaourova "*Undecimated wavelet transforms for image denoising*", November 19, 2002.


 