s*gα - totally continuous functions in topological spaces

V.R.Karuppayal [#1], N.Nandhini [*2]

1. Associate Professor, Department of Mathematics, Kongunadu Arts and Science College
Coimbatore-29,Tamilnadu, India.
2. M.phil Scholar, Department of Mathematics, Kongunadu Arts and Science College
Coimbatore-29,Tamilnadu, India.
[1] V.R.karuppayal@gmail.com
[2] nandhudec1@gmail.com

Abstract: **In this paper we introduced s*gα - totally continuous functions and investigate their properties. Also we derive some basic properties of s*gα - totally continuous functions and three new class s*gα -T_1, s*gα-T_2, s*gα -T_0.**

Keywords: s*gα - totally continuous functions.

1.INTRODUCTION

N.Levine [9] introduced the class of s*gα -continuous functions. Jain [8] introduced totally continuous. T.M. Nour [12] introduced the concept of totally s*gα-continuous functions as a generalization of totally continuousnfunctions and several properties of totally s*gα-continuous .S.Ayawarya[1] introduced a new class of sets namely s*gα-closed sets in topological spaces and derive its properties , and also find the relationship between s*gα-closed sets and other sets. In this paper we introduced the s*gα - totally continuous functions . Also we derive some basic properties of s*gα - totally continuous functions and three new class s*gα -T_1, s*gα -T_2, s*gα -T_0.

II. PRELIMINARIES

Throughout this paper (X, τ), (Y,σ) and (Z, γ) represent non-empty topological spaces. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A in X respectively. The power set of X is denoted by P(X).

Definition 2.1: A subset A of a space (X, τ) is called

1) a generalized closed (briefly g - closed) set [9] if cl(A) $\subseteq$ U and U is open in (X,τ) the complement of a g - closed set is called a g - open,

2) a semi-generalized closed(briefly sg-closed) set [2] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ) ; the complement of sg-closed set is called sg - open set,

3) a generalized semi-closed (briefly gs-closed) set [4] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ) ; the complement of gs-closed set is gs-open set ,

4) a s*gα-closed set [1] scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is *gα-open (X, τ),

5) a generalized α-closed set (briefly gα-closed) [5] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (X, τ),

Definition 2.2: A subset A of a space (X, τ) is called

1) s*gα-continuous [1] if the inverse image of each open subset of Y is s*gα-open in X.

2) totally-continuous [9] if the inverse image of each open subset of Y is clopen subset of X.

3) strongly-continuous [14] if the inverse image of each subset of Y is clopen subset of X.

4) totally s*gα-continuous [13] if the inverse image of each open subset of Y is s*gα-clopen subset of X.

5) strongly s*gα - continuous [13] if the inverse image of each subset of Y is s*gα - clopen subset of X.

6) pre s*gα - open [3] if the inverse image of every s*gα - open set in X is s*gα - open in Y.

***Definition* 2.3** The topological space (X, τ) is said to be

1) a $T_{½}^{*}$ space [10] if every g^{*}- closed set is closed.

2) a T_b space [6] if every gs - closed set is closed.

3) a T_c space [11] if every gs - closed set is g^{*}-closed.

4) a ${}_{\alpha}T_b$ space [5] if every αg - closed set is closed.

5) a ${}_{\alpha}T_c$ space [5] if every αg - closed set is g^{*}-closed.

6) a ${}_{\alpha}T_{½}^{**}$ space [16] if every *gα - closed set is closed.

7) a T_c^{**} space [17] if every gs - closed set is *gα - closed.

8) a ${}_{\alpha}T_c^{**}$ space [17] if every αg-closed set is *gα - closed.

Definition 2.4 Let A be a subset of a space (X, τ).

1) The set ∩{U ϵ τ : A ⊂ U} is called the kernesl of A is denoted by ker(A).

2) The set ∩{F ⊂ X : A ⊆ F,F is s*gα-closed} is called the s*gα-closure of A and is denoted by s*gαcl(A).

3) The set ∪{F ⊂ X : F ⊆ A,F is s*gα-open} is called the s*gα-interior of A and is denoted by s*gαint(A).

4) Let f: (X, τ) → (Y, σ) be a function, the subset {(x,f(x)) : x ϵ X} ⊆ X × Y is called the graph of f and is denoted by Gr(t).

5) Let f: (X, τ) → (Y, σ) be a function, the subset {(x,f(x)) : x ϵ X} ⊆ X × Y is called the graph of f and is denoted by Gr(t).

DEFINITION 2.5

1) a locally indiscrete [10] if each open subset of X is closed in X;

2) s*gα-connected [15] if X cannot be written as a disjoint union of two non-empty s*gα-open;

3) ultra normal [13] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets;

4) weakly hausdorff [13] if each element of X is an intersection of regular closed sets;

5) ultrahausdorff [15] if for each pair of distinct points x and y in X, there exist clopen sets A and B containing x and y, respectively, such that A∩B = ϕ;

III s*gα-totally continuous functions in topological spaces

***Definition* 3.1**

Let X be a topological space and x ∈ X. Then the set of all points y in X such that x and y cannot be separated by s*gα-separation of X is said to be the quasi s*gα-component of x. A quasi s*gα - component of a point x in a space X means the intersection of all s*gα-clopen sets containing x.

***Definition* 3.2**

A function f: X → Y is said to be s*gα-totally continuous function if the inverse image of every s*gα-open subset of Y is clopen in X.

***Example* 3.3**

Let X = Y = {a, b, c},τ = {X, ϕ , {a} , {b, c}} and σ = {Y, ϕ, {a},{b},{a, b}}Then S*GOα(Y) = {Y, ϕ, {a}, {a, b}, {a, c}}. Define f(a) = a, f(b) = b and f(c) = c. Clearly the inverse image of each s*gα - open is clopen in X. Therefore f is a s*gα - totally continuous function.

Theorem 3.4

A function f: X → Y is s*gα-totally continuous if and only if the inverse image of every s*gα-closed subset of Y is clopen in X.

Proof

Let F be any s*gα-closed set in Y. Then Y − F is s*gα - open set inY. By definition $f^{-1}(Y - F)$ is clopen in X. That is X - $f^{-1}(F)$ is clopen inX. This implies $f^{-1}(F)$ is clopen in X. if V is s*gα - open in Y , then Y−V is s*gα - closed in Y .By hypothesis, $f^{-1}(Y - V) = X - f^{-1}(V)$ is clopen in X, which implies $f^{-1}(V)$ is clopen in X. Thus, inverse image of every s*gα - open set in Y is clopen in X. Therefore f is s*gα - totally continuous function.

Theorem 3.5

Every s*gα - totally continuous function is a totally continuous function.

Proof

Suppose f: X → Y is s*gα - totally continuous and U is any open subset of Y. Since every open set is s*gα - open, U is s*gα - open in Y and f: X → Y is s*gα - totally continuous, it follows $f^{-1}(U)$ is clopen in X. Thus inverse image of every open set in Y is clopen in X. Therefore the function f is totally continuous. The converse of the above theorem need not be true.

***Example* 3.6**

Let X =Y = {a, b, c}, τ = {X, ϕ, {a},{b, c}} and σ = {Y, ϕ, {a}}.Then S*Gα(Y) = {Y, ϕ, {a},{b},{c},{a, c}}. Define f: (X, τ) → (Y, σ) asf (a) = a, f (b) = b, f(c) =c. Clearly the inverse image of every open set is clopen. Therefore f is totally continuous. But f is not s*gα - totally continuous, because for the s*gα-open set {a, b}, f^{-1}{a, c} = {a, c} is not clopen in X.

Theorem 3.7

Every s*gα - totally continuous function is totally s*gα - continuous.

Proof

Suppose f: X → Y is s*gα - totally continuous function and A is any open set in Y. Since every open set is s*gα - open and f: X → Y is s*gα - totally continuous, it follows that $f^{-1}(A)$ is clopen and hence s*gα-clopen in X. Thus the inverse image of each open set in Y is s*gα-clopen in X. Therefore f is totally s*gα-continuous. The converse of the above need not be true.

***Example* 3.8**

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {a}}. S*Gα(X) = {X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}} and S*Gα (Y) = {Y, ϕ, {a},{b},{c},{a, b},{a, c}}. Define f: (X, τ) → (Y, σ) as f(a) = a and f(b) = f(c) = b. Clearly the inverse image of every open set is s*gα-clopen. Therefore f is totally s*gα-continuous. But f is not s*gα-totally countinuous, because for the s*gα-open set {a}, f^{-1}{a,c} = {a,c} is not clopen in X.

Theorem 3.9

Let f: X → Y be a function, where X and Y are topological spaces. Then following are equivalent:

(i) f is s*gα-totally continuous.

(ii) for each x ∈ X and each s*gα-open set V in Y with f (x) ∈ V , there is a clopen set U in X such that x ∈ Uand f (U) ⊂ V .

Proof

(i)⇒(ii)

Suppose f: X → Y is s*gα-totally continuous and V be any s*gα-open set in Y containing f(x) so that x ∈ $f^{-1}(V)$. Since f is s*gα - totally continuous, $f^{-1}(V)$ is clopen in X. Let U = $f^{-1}(V)$, then U is clopen set in X and x ∈ U. Also f(U) = $f(f^{-1}(V)) \subset V$. This implies f(U)) ⊂ V .

(ii)⇒(i)

Let V be s*gα-open in Y. Let x ∈ $f^{-1}(V)$ be any arbitrary point.This implies f(x) ∈ V. Therefore by (ii) there is a clopen set $f(G_x) \subset X$ containing x such that $f(G_x) \subset V$, which implies $G_x \subset f^{-1}(V)$. We have $x \in G_x \subset f^{-1}(V)$. This implies $f^{-1}(V)$ is clopen neighbourhood of x. Since x is arbitrary, it implies $f^{-1}(V)$ is clopen neighbourhood of each of its points. Hence it is clopen set in X. Therefore f is s*gα-totally continuous.

***Definition* 3.11**

A space X is said to be s*gα–T_1 iff for each pair of distinct points x and y in X, these exists s*gα-open sets U and V containing x respectively, such y∉U and x∉V.

Theorem 3.12

Let f: X → Y be a s*gα - totally continuous function from a space X into a s*g space Y . Then f is constant on each quasi s*gα - component of X.

Proof

Let a and b be two points of X that lie in the same quasi s*gα - component of X. Then f(a) and f(b) are elements in Y . Assume f(a) = α ≠ β= f(b). Since Y is semi-T1, {α} is s*gα - closed in Y and so Y - {α} is s*gα - open. Since f: X → Y is s*gα - totally continuous $f^{-1}(\{\alpha\})$ and $f^{-1}(Y - \{\alpha\})$ are disjoint clopen subsets of X. Further a ∈ $f^{-1}(\{\alpha\})$ and b ∈ $f^{-1}(Y - \{\alpha\})$,which is a contradiction in view of the fact that b belongs to quasi s*gα -component of a and hence b must belong to every clopen set containing. Hence the result.

***Definition* 3.13**

s*gα -connected if X cannot be written as a disjoint of two non-empty s*gα -open.

Theorem 3.14

If f : X → Y is s*gα - totally continuous function from an s- connected space X onto any space Y, then Y is an indiscrete space.

Proof

Suppose f: X → Y is a s*gα - totally continuous function from an s- connected space X onto any space Y. If possible, suppose Y is not indiscrete. Let A be a proper non empty s*gα - open subset of Y. Then $f^{-1}(A)$ is a proper non - empty clopen and hence s*gα - clopen subset of X. This implies $f^{-1}(A)$ is a proper non - empty s*gα - open subset of X, which is a contradiction to the fact that X is s - connected. Therefore Y must be indiscrete.

Theorem 3.15

The composition two s*gα - totally continuous functions is s*gα - totally continuous.

Proof

Let f: X → Y and g: Y → Z be any two s*gα - totally continuous functions. Let V be a s*gα - open set in Z. Since g is s*gα - totally continuous $g^{-1}(V)$ is clopen and hence open in Y. Since every open set is s*gα - open, $g^{-1}(V)$ is s*gα - open in Y. Further, since f is s*gα - totally continuous,

$f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is clopen in X.

Hence g∘f : X → Z is s*gα - totally continuous.

Theorem 3.16

Let f: X → Y be s*gα-totally continuous and g: Y → Z be any function. Then g∘f: X → Z is s*gα-totally continuous if and only if g is irresolute.

Proof

Let g: Y → Z be irresolute. let g∘f : X → Z be semi-totally continuous. Let V be semiopen set in Z. Since g∘f : X → Z is s*gα - totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in X. Since f is s*gα-totally continuous, $g^{-1}(V)$ is s*gα-open in Y. Thus the inverse image of each s*gα-open set in Z is s*gα-open inY. Hence g is irresolute.

***Definition* 3.17**

Let f: X → Y be a function. Then the graph function of f is defined by g(x) = (x, f(x)) for each x ∈ X.

Theorem 3.18

A function f: X → Y is s*gα-totally continuous, if its graph function is s*gα-totally continuous.

Proof

Let g: X → X × Y be the graph function of f: X → Y . Suppose g is s*gα-totally continuous and F be a s*gα-open set in Y. Then X × F is a s*gα-open set of X × Y. Since g is s*gα-totally continuous, $g^{-1}(X \times F) = f^{-1}(F)$ is clopen in X. Thus the inverse image of every s*gα-open set in Y is clopen in X. Therefore f is s*gα-totally continuous. Let $\{X_\lambda : \lambda \in \wedge\}$ be a family of topological spaces. Then the product space of $\{X_\lambda : \lambda \in \wedge\}$ is denoted by $\Pi\{X_\lambda : \lambda \in \wedge\}$ or simply ΠX_λ.

Theorem 3.19

If a function $f: X \to \Pi Y_\lambda$ is s*gα-totally continuous, then $p_\lambda \circ f: X \to Y_\lambda$ is s*gα-totally continuous for each $\lambda \in \wedge$, where p_λ is the projection of ΠY_λ on to Y_λ.

Proof

For $\lambda \in \wedge$, suppose V_λ is any s*gα-open set in Y_λ. Then $p^{-1}_\lambda (V_\lambda)$ is s*gα-open in ΠY_λ. Since f is s*gα-totally continuous, $f^{-1}(p_\lambda^{-1}(V_\lambda)) = (p_\lambda \circ f)^{-1}(V_\lambda)$ is clopen in X. Therefore $f: X \to Y_\lambda$ is s*gα-totally continuous.In the sequel, the relationships between s*gα-totally continuous functions and separation axioms are investigated.

Theorem 3.20

If f: X → Y is s*gα-totally continuous injection and Y is s*gα–T_0, then X is ultra-Hausdorff.

Proof

Let a and b be any pair of distinct points of X and f be injective. Then f(a) ≠ f(b) in Y . Since Y is s*gα–T_0, there exists a s*gα - open set U containing say f(a) but not f(b). Then, we have a ∈ f^{-1}(U) and b ∉ f^{-1}(U). Since f is s*gα - totally continuous f^{-1}(U) is clopen in X. Also a ∈ f^{-1}(U) and b ∈ X - f^{-1}(U). This implies every pair of distinct points of X can be separated by disjoint clopen set s in X. Therefore X is ultra - Hausdorff.

Theorem 3.21

If f: X → Y is s*gα - totally continuous injection and Y is s*g-T_2, then X is ultra - Hausdorff.

Proof

Let x_1, x_2 ∈ X and $x_1 \neq x_2$. Then, since f is injective, $f(x_1) \neq f(x_2)$ in Y. Further, since Y is s*gα-T_2, there exist V_1 and V_2 ∈ SO(Y) such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$. This implies $x_1 \in f^{-1}(V_1)$ and $x_2 \in f^{-1}(V_2)$. Since f is s*gα-totally continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are clopen sets in X.
Also $f^{-1}(V_1) \cap f^{-1}(V_1) = f^{-1}(V_1 \cap V_2) = \phi$. Thus every two distinct points of X can be separated by disjoint clopen sets. Therefore X is ultra - Hausdorff.

Theorem 3.22

If f: X → Y is s*gα - totally continuous, closed injection and Y is s - normal, then X is ultra - normal.

Proof

Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y . Since Y is s - normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint s*gα - open sets V_1 and V_2 respectively. Therefore we obtain, $F_1 \subset f^{-1}(V_1)$ and $F_2 \subset f^{-1}(V_2)$. Since f is s*gα - totally continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are clopen sets in X. Also, $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = \phi$. Thus each pair of non-empty disjoint closed sets in X can be separated by disjoint clopen sets in X.
Therefore X is ultra - normal.

Theorem 3.23

If f: X → Y is s*gα - totally continuous surjection and X is connected then Y is s - connected.

Proof

Suppose Y is not s-connected. Let A and B form disconnection of Y .Then A and B are s*gα - open sets in Y and Y = A∪ B where A∩B = ϕ. Also X = f^{-1}(Y) = f^{-1}(A ∪ B) = f^{-1}(A) ∪ f^{-1}(B), where f^{-1}(A) and f^{-1}(B) are non-empty clopen sets in X, because f is s*gα-totally continuous. Further
f^{-1}(A) ∩ f^{-1}(B) = f^{-1}(A ∩ B) = ϕ. This implies X is not connected, which is a contradiction. Hence Y is s - connected.

Theorem 3.24

Let f: X → Y be a totally continuous, closed injection. If Y is s-regular then X is ultra-regular.

Proof

Let F be a closed set not containing x. Since f is closed, we have f(F) is a closed set in Y not containing f(x). Since Y is s-regular, there exists disjoint s*gα-open sets A and B such that f(x) ∈ A and f(F) ⊂ B, which implies x ∈ f^{-1}(A) and F ⊂ f^{-1}(B), where f^{-1}(A) and f^{-1}(B) are clopen sets in X because f is totally continuous. Moreover, since f is injective, we have f^{-1}(A) ∩ f^{-1}(B) = f^{-1}(A∩B) = f^{-1}(ϕ) = ϕ. Thus, for a pair of a point and a closed set not containing the point, they can be separated by disjoint clopen sets. Therefore X is ultra-regular.

Theorem 3.25

If f: X → Y is s*gα-totally continuous injective s*gα-open function from a clopen normal space X onto a space Y, then Y is s*gα-normal.

Proof

Let F_1 and F_2 be any two disjoint s*gα-closed sets in Y. Since f is s*gα-totally continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are clopen subsets of X. Take U = $f^{-1}(F_1)$ and V = $f^{-1}(F_2)$. Since f is injective U ∩V = $f^{-1}(F_1) \cap f^{-1}(F_2) = f^{-1}(F_1 \cap F_2) = f^{-1}(\phi) = \phi$. Since X is clopen normal there exist disjoint open sets A and B such that U ⊂ A and V ⊂ B. This implies F_1 = f(U) ⊂ f(A) and F_2 = f(V) ⊂ f(B). Further, since f is injective s*gα-open, f(A) and f(B) are disjoint s*gα - open sets. Thus, each pair of disjoint s*gα-closed sets can be separated by disjoint s*gα-open sets. Therefore Y is s*gα-normal.

***Definition* 3.26**

A function f: X → Y is said to be s*gα - totally open if the image of every s*gα-open set in X is clopen in Y.

Theorem 3.27

If a bijective function f : X → Y is s*gα - totally open, then the image of each s*gα - closed set in X is clopen in Y .

Proof

Let F be a s*gα - closed set in X. Then X-F is s*gα - open in X. Since f is s*gα - totally open, f(X −F) = Y - f(F) is clopen in Y . This implies f(F) is clopen in Y .

Theorem 3.28

A surjective function f : X → Y is s*gα - totally open if and only if for each subset B of Y and for each s*gα -closed set U containing $f^{-1}(B)$, there is a clopen set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof

Suppose f: X → Y is a surjective s*gα-totally open function and $B \subset Y$. Let U be s*gα-closed set of X such that $f^{-1}(B) \subset U$. Then V = Y -f(X − U) is clopen subset of Y containing B such that $f^{-1}(V) \subset U$. On the other hand, suppose F is a s*gα - closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ and X − F is s*gα - open. By hypothesis, there exists a clopen set V of Y such that $Y - f(F) \subset V$, which implies $f^{-1}(V) \subset X - F$. Therefore $F \subset X - f^{-1}(V)$. Hence $Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$. This implies, f(F) = Y − V , which is clopen in Y . Thus, the image of a s*gα - openset in X is clopen in Y. Therefore f is a s*gα - totally open function.

Theorem 3.29

The composition of two s*gα - totally open functions is again s*gα - totally open.

Proof

Suppose f: X → Y and g: Y → Z are any two s*gα - totally open functions. Then their composition is $g \circ f$: X → Z. Let V be a s*gα - open set in X. Consider $(g \circ f)(V)$= g(f(V)). Since f is s*gα - totally open, f(V) is clopen in Y . Hence it is open in Y. But every open set is s*gα - open, which implies f(V) is s*gα - open in Y . Since g is s*gα - totally open, g(f(V)) is clopen in Z. Thus, the image of each s*gα - open set in X is clopen in Z. Therefore $g \circ f$: X → Z is s*gα - totally open.

***Theorem* 3.30**

If f: X → Y is s*gα - totally continuous and s*gα - totally closed surjection from an s - normal space X to a space Y , then Y is ultra - Hausdorff.

Proof

Let A and B be disjoint closed sets of Y. Since f: X → Y is s*gα-totally continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are clopen hence closed sets in X. Since X is s - normal, there exist disjoint s*gα-open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. There are clopen sets G and H such that $A \subset G$, $B \subset H$ and $f^{-1}(G) \subset U$, $f^{-1}(H) \subset V$. Then we have, $f^{-1}(G) \cap f^{-1}(H) \subset U \cap V = \phi$, which implies $f^{-1}(G \cap H) \subset \phi$, which implies $G \cap H = \phi$. Thus every pair of non-empty disjoint closed sets can be separated by disjoint clopen sets. Therefore Y is ultra-Hausdorff.

***Definition* 3.31**

A function f: (X, τ) → (Y, σ) is called totally s*gα-continuous if the inverse image of every open set of (Y, σ) is a s*gα-clopen subset of (X, τ).

Theorem 3.32

Every totally s*gα-continuous function is s*gα-continuous.

Proof

Let V be a open set of (Y, σ).Since f is totally s*gα-continuous, $f^{-1}(V)$ is s*g-closed in (X, τ).Since every closed set is s*gα-closed. Hence $f^{-1}(V)$ is s*gα-closed in (X,τ). Therefore f is s*gα-continuous.

***Example* 3.33**

Let X = Y = {a, b, c} with τ = {ϕ, X, {a}} and σ = {ϕ, Y,{a},{a, b}}, σ^c= {ϕ, Y,{c},{b, c}}.The function f: (X, τ) → (Y, σ) be defined as f(a) = b, f(b) = a and f(c) = c. The s*gα - open sets of (X, τ) are ϕ, X,{a},{b},{c},{a, c},{a, b}.Thus f is s*gα-continuous but not totally s*gα-continuous. Since the inverse image of the open set f^{-1}{a} = {a} in (Y,σ) is not s*gα-open in (X, τ).

***Definition* 3.35**

A topological space (X, τ) is said to be s*gα-connected union of two non - empty disjoint s*gα - open sets.

***Theorem* 3.36**

A space (X, τ) is s*gα - connected if and only if every totally s*gα - continuous function from a space (X, τ) into any T_o space (Y, σ) is a constant map.

Proof

Necessity part

Suppose f: (X, τ) → (Y, σ) is a totally s*gα - continuous function, where (Y, σ) is a T_o space. On the contrary if we suppose that f is not a constant map, then we can select two points x and y in X such that f(x) → f(y).Since (Y,σ) is a T_o space and f(x) and f(y) are distinct points of Y, then there exists an open set G in (Y, σ) containing f(x) but not f(y).Since f is totally s*gα - continuous function, then $f^{-1}(G)$ is s*gα

- clopen subsets of (X, τ). Clearly x f^{-1}(G) and f^{-1}(G). Now X = f^{-1}(G) ⊂ (f^{-1}(G)), which is the union of two non - empty s*gα - open subsets of X. Thus X is not a s*gα - connected space. A contradiction.

Sufficiency part

Suppose (X, τ) is not a s*gα - connected space. Then there exists a proper non-empty s*gα - clopen subset A of X. Let Y = {a, b} and X ={X, ϕ,{a},{b}}.Define f: (X, τ) → (Y, σ) by f(x) = a for any x and f(x) = b for any x .Clearly f is a non - constant and totally s*gα - continuous map. Clearly (Y, σ) is a T_o space. Thus we have produced a non - constant totally s*gα - continuous function from (X, τ) into the T_o space (Y, σ).A contradiction.

CONCLUSION

In this paper we introduced a new class of sets namely s*gα -totally continuous function in topological spaces and using this sets we introduced three new class s*gα -T_1, s*gα -T_2, s*gα -T_0.

REFERENCES

[1] K.Ayswarya, A Study On Group Structure of closed sets in topological space, M.phi dissertation Kongunadu Arts and Science College coimbatore(2014)

[2] P. Bhattacharyya and B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29(3) (1987), 375-382.

[3] S. G. Crossley and S. K. Hildebrand, Semi-Topological properties, Fund.

[4] R.Devi, Studies On generalizations of closed maps and homeomorphisms in topological spaces, Ph.D.Thesis, Bharathiar University, Coimbatore(1994).

[5] R. Devi, K. Balachandran, H. Maki and ,Generalized □-closed maps and □-generalized closed maps, Indian J.Pure.Appl.Math.,29(1)(1998),37-49.

[6] R. Devi, H. Maki and K. Balachandran, Semi-generalized homeomorphisms and generalized Semi-homeomorphisms in topological spaces, Indian J.Pure.Appl.Math.,26(3)(1995),271-284.

[7] J. Dontchev, On generalizing semi-preopen sets ,Mem. Fac. Sci. Kochi Ser.A,Math.,16 (1995),35-48.

[8] R. C. Jain, The role of regularly open sets in general topology, Ph.D. thesis, Meerut University, Institute of advanced studies, Meerut-India, (1980).

[9] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(2) (1970), 89-96.

[10] T. Nieminen, On ultrapseudo compact and related spaces, Ann. Acad.Sci. Fenn. Ser. A I Math., 3 (1977), 185 – 205.

[11] O. Njastad ,On some classes of nearly open sets, Pacific J.Math.,15(1965), 961-970.

[12] T.M.Nour, Totally semi-continuous functions, Indian J. Pure Appl.Math.,26(7)(1995),675-678.

[13] T. Soundararajan, Weakly Housdorff spaces and the cardinality of topological spaces, In : General topology and its relation to modern analysis and algebra, III Proc. Conf. Kanpur, 1968, Academia, Prague,(1971), 301-306.

[14] M. Stone, Applications of the theory of boolean rings to general topology, Trans. Amer. Math. Soc., 41 (1937), 374.

[15] R. Staum, The algebra of bounded continuous functions into a nonarchimedean field, Pacific J. Math., 50 (1974), 169 – 185.

[16] M .K .R .S.Veerakumar, Between g^* - closed sets and g-closed sets, Antartica J.Math.,**3**(1)(2006),43-65.

[17] M.Vigneshwaran, A study on *g□–closed sets in Topological spaces, M.Sc. Dissertation, Kongunadu Arts and Science (Autonomous),Coimbatore(2006).

[18] M.Vigneshwaran and R. Devi, On G□o-kernel in the digital plane, International Journal of Mathematical Archive-**3**(6)(2012),2358-2373.

