On Vertex Polynomial of Snake Graphs

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***Abstract* - Let $G = (V, E)$ be a graph. The vertex polynomial of the graph $G = (V, E)$ is defined as $V(G, x) = \sum_{k=0}^{\Delta(G)} v_k x^k$, where $\Delta(G) = \max\{d(v)/v \epsilon V\}$ and v_k is the number of vertices of degree k. In this paper we seek to find the vertex polynomial of snake graphs.**

***Keywords*- Triangular Snake, Quadrilateral snake, Double Triangular Snake, Double Quadrilateral Snake, Vertex Polynomial, Union, Sum.**

1. INTRODUCTION

In a graph $G = (V, E)$, we mean a finite undirected, non-trivial graph without loops and multiple edges. The vertex set is denoted by V and the edge set by E. For $v \epsilon V$, $d(v)$ is the number of edges incident with v. The maximum degree of $G$is defined as $\Delta(G) = \max\{d(v)/v \epsilon V\}$. For terms not defined here, we refer to Frank Harary [2]. The graph $G = (V, E)$ is simply denoted by G. Let G_1 and G_2be two graphs, the union $G_1 \cup G_2$ is defined to be (V, E) where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$, the sum $G_1 + G_2$ is defined as $G_1 \cup G_2$ together with all the lines joining points of V_1 to V_2.

2. VERTEX POLYNOMIAL OF TRIANGULAR SNAKE(T_n), THEIR UNION AND THEIR SUM

Definition: 2.1

A Triangular Snake T_n is obtained from a path $u_1 u_2 \dots u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$. That is every edge of a path is replaced by a triangle C_3.

Theorem: 2.2

Let G be a Triangular snake(T_n). Then the vertex polynomial of G is given by

$$V(G, x) = (n-2)x^4 + (n+1)x^2, n \geqslant 2.$$

Proof:

Let G be a Triangular snake(T_n). Then G has order $2n - 1$. Since Triangular snake(T_n) is obtained from a path P_n in which each edge of P_n is replaced by C_3, we get $n - 2$ vertices have degree 4 and $n + 1$ vertices have degree 2. Hence $V(G, x) = \quad (n-2)x^4 + (n+1)x^2, n \geqslant 2$.

Example: 2.3

Consider the Triangular snake T_3. Then the corresponding graph is illustrated as follows;

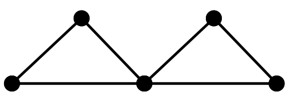

Figure: 1

Here, $V(G, x) = x^4 + 4x^2$.

Result: 2.4

(i) Let G be a Triangular snake(T_n). Then the vertex polynomial of $\zeta = G \cup G \cup \dots \cup G\ (m\ times)$ is given by

$$V(\zeta, x) = m(n-2)x^4 + m(n+1)x^2, n \geqslant 2.$$

(ii) Let G be a Triangular snake(T_n). Then the vertex polynomial of mG is given by

$$V(mG, x) = m(n-2)x^{4+(m-1)(2n-1)} + m(n+1)x^{2+(m-1)(2n-1)}, n \geqslant 2.$$

3. VERTEX POLYNOMIAL OF QUADRILATERAL SNAKE(Q_N), THEIR UNION AND THEIR SUM.

Definition: 3.1

A Quadrilateral Snake Q_n is obtained from a path $u_1 u_2 \dots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and w_i. That is every edge of a path is replaced by a cycle C_4.

Theorem: 3.2

Let G be a Quadrilateral snake(Q_n). Then the vertex polynomial of G is given by

$$V(G, x) = (n-2)x^4 + 2nx^2, n \geqslant 2.$$

Proof:

Let G be a Quadrilateral snake(Q_n). Then G has order $3n - 2$. Since Quadrilateral snake(Q_n) is obtained from a path P_n in which each edge of P_n is replaced by C_4, we get $n - 2$ vertices have degree 4 and $2n$ vertices have degree 2. Hence $V(G, x) = \quad (n-2)x^4 + 2nx^2, n \geqslant 2$.

Example: 3.3

Consider the Quadrilateral snake(Q_3). Then the corresponding graph is explained as follows;

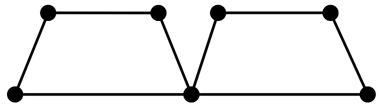

Figure: 2

Here, $V(G, x) = x^4 + 6x^2$.

Result: 3.4

(i) Let G be a Quadrilateral snake(Q_n). Then the vertex polynomial of $\zeta = G \cup G \cup \dots \cup G\ (m\ times)$ is given by

$$V(\zeta, x) = m(n-2)x^4 + 2nmx^2, n \geqslant 2.$$

(ii) Let G be a Quadrilateral snake(Q_n). Then the vertex polynomial of mG is given by

$$V(mG, x) = m(n-2)x^{4+(m-1)(3n-2)} + 2nmx^{2+(m-1)(3n-2)}, n \geqslant 2.$$

4. VERTEX POLYNOMIAL OF DOUBLE TRIANGULAR SNAKE $D(T_N)$, THEIR UNION AND THEIR SUM.

Definition: 4.1

The Double Triangular Snake $D(T_n)$ consists of two Triangular snakes that have common path.

Theorem: 4.2

Let G be a Double Triangular snake $D(T_n)$. Then the vertex polynomial of G is given by
$V(G, x) = (n-2)x^6 + 2x^3 + (2n-2)x^2, n \geqslant 2.$

Proof:

Let G be a Double Triangular snake with $3n-2$. From the definition of Double Triangular snake, $n-2$ vertices have degree 6, 2 vertices have degree 3 and $2n-2$ vertices have degree 2 gives the required result.

Example: 4.3

Consider the Double Triangular snake $D(T_5)$. Then the corresponding graph is illustrated as follows;

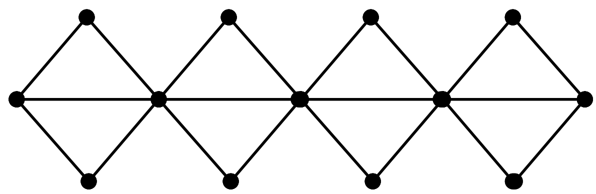

Figure: 3

Here, $V(G, x) = 3x^6 + 2x^3 + 8x^2$.

Results: 4.4

(i) Let G be a Double Triangular snake $D(T_n)$. Then the vertex polynomial of $\zeta = G \cup G \cup \ldots \cup G\ (m\ times)$ is given by $V(\zeta, x) = m(n-2)x^6 + 2mx^3 + m(2n-2)x^2, n \geqslant 2.$

(ii) Let G be a Double Triangular snake $D(T_n)$. Then the vertex polynomial of mG is given by

$$V(mG, x) = m(n-2)x^{6+(m-1)(3n-2)} + 2mx^{3+(m-1)(3n-2)} + m(2n-2)x^{2+(m-1)(3n-2)}, n \geqslant 2.$$

5. VERTEX POLYNOMIAL OF DOUBLE QUADRILATERAL SNAKE $D(Q_N)$, THEIR UNION AND THEIR SUM.

Definition: 5.1

The Double Quadrilateral Snake $D(Q_n)$ consists of two Quadrilateral snakes that have common path.

Theorem: 5.2

Let G be a Double Quadrilateral Snake $D(Q_n)$. Then the vertex polynomial of G is given by
$V(G, x) = (n-2)x^6 + 2x^3 + 4(n-1)x^2, n \geqslant 2.$

Proof:

Let G be a Double Quadrilateral Snake with order $5n-4$. Since, the definition of Double Quadrilateral snake, $n-2$ vertices have degree 6, 2 vertices have degree 3 and $4(n-1)$ vertices have degree 2 gives that $V(G, x) = (n-2)x^6 + 2x^3 + 4(n-1)x^2, n \geqslant 2.$

Example: 5.3

Consider the Double Quadrilateral snake $D(Q_4)$. Then the corresponding graph is illustrated as follows;

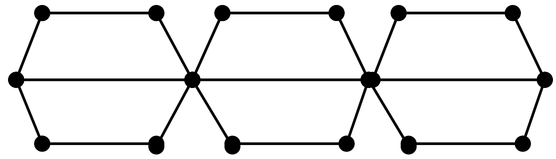

Figure: 4

Here, $V(G, x) = 2x^6 + 2x^3 + 12x^2$.

Results: 5.4

(i) Let G be a Double Quadrilateral snake $D(Q_n)$. Then the vertex polynomial of $\zeta = G \cup G \cup \ldots \cup G\ (m\ times)$ is given by $V(\zeta, x) = m(n-2)x^6 + 2mx^3 + 4m(n-1)x^2, n \geqslant 2.$

(ii) Let G be a Double Quadrilateral snake $D(Q_n)$. Then the vertex polynomial of mG is given by $V(mG, x) = m(n-2)x^{6+(m-1)(5n-4)} + 2mx^{3+(m-1)(5n-4)} + m(2n-2)x^{2+(m-1)(5n-4)}, n \geqslant 2.$

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