# Fuzzy Optimization Inventory Model with Allowable Shortage Using Hexagonal Fuzzy Numbers

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***Abstract:*** **In this paper, a fuzzy inventory model for allowable shortage is derived. The objective is to find the optimal total cost and optimal order quantity for the given inventory model. Ordering cost, holding cost and shortage cost are taken as hexagonal fuzzy numbers. For defuzzification, signed distance method is used. Numerical example is illustrated for the proposed model.**

***Key Words:*** **Hexagonal fuzzy numbers, allowable shortage, crisp and fuzzy inventory model, defuzzification.**

## I. INTRODUCTION

Inventory control is a recently developing research area, which has a huge number of practical applications. In earlier days, the uncertainties of inventory models were handled using probability theory. The first quantitative treatment of inventory was the EOQ model. The model was developed and applied in both academics and industries. Later, it helped to develop many other inventory systems. In some situations, uncertainties are due to fuzziness, primarily introduced by Zadeh [9] is applicable. Later on, so many researchers worked on these areas. Then, many applications on fuzzy set theory can be found.

In EOQ model, the order size, which minimizes the sum of annual total cost of inventory is derived. Thus, EOQ model is useful in approximations and in finding solutions for real life problems. Urgeletti[6] developed EOQ model using triangular fuzzy numbers. Chan [1] developed using trapezoidal fuzzy numbers to find the total cost. Vujosevic[7] used trapezoidal fuzzy numbers to find the total cost with backorder.

In this paper, an inventory model using hexagonal fuzzy number is developed for holding cost, ordering cost and shortage cost. For defuzzification, signed distance method is used. Due to irregularities or physical properties of the material, parameters cannot be considered as variables all the time .In such situations, fuzzy concepts are applied. Shortage is allowed and is completely backlogged. An algorithm is developed to find the optimal order quantity and also for minimizing the total cost. signed distance method is applied to defuzzify the fuzzy total cost and then solve the optimal order quantity.

## II. DEFINITIONS AND METHODOLOGY

### 2.1. Fuzzy set:

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$ .Here $\mu_{\tilde{A}}:X\to[0,1]$ is a mapping called the membership value of $x\in X$ in a fuzzy set $\tilde{A}$.

### 2.2. Convex Fuzzy Set:

A fuzzy set $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}\subseteq X$ is called convex fuzzy set if all $A_x$ are convex sets. (i.e.) For every element $x_1\in A_\alpha$ and $x_2\in A_\alpha$ for every $\alpha\in[0,1]$, $\lambda x_1+(1-\lambda)x_2\in A_\alpha \forall\lambda\in[0,1]$. Otherwise the fuzzy set is called non-convex fuzzy set.

### 2.3. Hexagonal Fuzzy Number:

A fuzzy number on $\tilde{A}_h$ is a hexagonal fuzzy number denoted by $\tilde{A}_h=(a_1,a_2,a_3,a_4,a_5,a_6)$ where $(a_1\le a_2\le a_3\le a_4\le a_5\le a_6)$ are real numbers satisfying $a_2-a_1\le a_3-a_2$ and $a_5-a_4\ge a_6-a_5$ and its membership function $\mu_{\tilde{A}_h}(x)$ is given as

 

$$\mu_{\tilde{A}_h}(x)=\begin{cases}0, x<a_1\\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), a_1\le x\le a_2\\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), a_2\le x\le a_3\\ 1, a_3\le x\le a_4\\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), a_4\le x\le a_5\\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), a_5\le x\le a_6\\ 0, x>a_6\end{cases}$$

**2.4. Remark:**

1. The Hexagonal fuzzy number $\tilde{A}_h$ becomes trapezoidal fuzzy number if $a_2-a_1=a_3-a_2$ and $a_5-a_4=a_6-a_5$.
2. The Hexagonal fuzzy numbers $\tilde{A}_h$ becomes non-convex fuzzy number if $a_2-a_1>a_3-a_2$ and $a_5-a_4<a_6-a_5$

**2.5. Equality of Two Hexagonal Fuzzy Numbers:**

A fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6)$ and $\tilde{B}=(b_1,b_2,b_3,b_4,b_5,b_6)$ are said to be equal (i.e.) $\tilde{A}=\tilde{B}$ iff $a_1=b_1, a_2=b_2, a_3=b_3, a_4=b_4, a_5=b_5, a_6=b_6$

**2.6. Symmetric Hexagonal Fuzzy Number:**

A hexagonal fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6)$ is said to be a symmetric Hexagonal fuzzy number, if $a_3-a_1=a_6-a_4$. Otherwise, the fuzzy number is called non-symmetric fuzzy number.

**2.7. Note:**

A Hexagonal fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6)$ is said to be non negative (non positive) (i.e.) $\tilde{A}\ge 0$ $(\tilde{A}\le 0)$ iff $a_1\ge 0 (a_6\le 0)$.

**2.8. New Arithmetic operations:**

The new arithmetic operations between hexagonal fuzzy numbers proposed are given below. Let us consider $\tilde{A}_1=(a_1,a_{2,}a_{3,}a_{4,}a_{5,}a_6)$ and $\tilde{A}_2=(b_{1,}b_{2,}b_{3,}b_{4,}b_{5,}b_6)$ be two hexagonal fuzzy numbers. Then,

The addition of $\tilde{A}_1$ and $\tilde{A}_2$ is

$$\tilde{A}_1(+)\tilde{A}_2=(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6)$$

The subtraction of $\tilde{A}_1$ and $\tilde{A}_2$ is

$$\tilde{A}_1(-)\tilde{A}_2=(a_1-b_6, a_2-b_5, a_3-b_4, a_4-b_3, a_5-b_2, a_6-b_1)$$

The multiplication of $\tilde{A}_1$ and $\tilde{A}_2$ is

$$\tilde{A}_1(\times)\tilde{A}_2=\left(\frac{a_1}{6}\sigma_b, \frac{a_2}{6}\sigma_b, \frac{a_3}{6}\sigma_b, \frac{a_4}{6}\sigma_b, \frac{a_5}{6}\sigma_b, \frac{a_6}{6}\sigma_b\right)$$

where $\sigma_b=(b_1+b_2+b_3+b_4+b_5+b_6)$

The division of $\tilde{A}_1$ and $\tilde{A}_2$ is

$$\tilde{A}_1(\div)\tilde{A}_2=\left(\frac{6a_1}{\sigma_b}, \frac{6a_2}{\sigma_b}, \frac{6a_3}{\sigma_b}, \frac{6a_4}{\sigma_b}, \frac{6a_5}{\sigma_b}, \frac{6a_6}{\sigma_b}\right)$$

if $\sigma_b\ne 0$ where $\sigma_b=(b_1+b_2+b_3+b_4+b_5+b_6)$

If $k\ne 0$ is a scalar, $k\tilde{A}$ is defined as

$$k\tilde{A}=\begin{cases}(ka_1, ka_2, ka_3, ka_4, ka_5, ka_6), k>0\\ (ka_6, ka_5, ka_4, ka_3, ka_2, ka_1), k<0\end{cases}$$

$$\sqrt{\tilde{A}}=\sqrt{a_1, a_{2,} a_{3,} a_{4,} a_{5,} a_6}$$
$$=\left(\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}, \sqrt{a_4}, \sqrt{a_5}, \sqrt{a_6}\right)$$

where $a_1, a_{2,} a_{3,} a_{4,} a_{5,} a_6$ are non-zero positive real numbers.

**2.9. Different types of hexagonal fuzzy numbers:**

- A Hexagonal fuzzy numbers $\tilde{A}=(a,a_2,a_3,a_4,a_5,a_6)$ is said to be a trapezoidal fuzzy number if
  $a_2-a_1=a_3-a_2 \quad a_5-a_4=a_6-a_5$
- A hexagonal fuzzy number $\tilde{A}=(a,a_2,a_3,a_4,a_5,a_6)$ is said to be a Convex hexagonal fuzzy number if
  $a_2-a_1\le a_3-a_2 \quad a_5-a_4\ge a_6-a_5$
- A hexagonal fuzzy number $\tilde{A}=(a,a_2,a_3,a_4,a_5,a_6)$ is said to be a Non convex hexagonal fuzzy number if
  $a_2-a_1>a_3-a_2 \quad a_5-a_4<a_6-a_5$

 

- A hexagonal fuzzy number $\tilde{A}=(a,a_2,a_3,a_4,a_5,a_6)$ is said to be a Symmetric hexagonal fuzzy number if $a_3-a_1=a_6-a_4$
- A hexagonal fuzzy number $\tilde{A}=(a,a_2,a_3,a_4,a_5,a_6)$ is said to be a Non symmetric Hexagonal fuzzy number type-1 $(l_s>r_s)$ if $a_3-a_1>a_6-a_4$
- A hexagonal fuzzy number $\tilde{A}=(a,a_2,a_3,a_4,a_5,a_6)$ is said to be a Non symmetric Hexagonal fuzzy number type-2 $(l_s<r_s)$ if $a_3-a_1<a_6-a_4$
- A hexagonal fuzzy number $\tilde{A}=(a,a_2,a_3,a_4,a_5,a_6)$ is said to be a Perfect symmetric hexagonal fuzzy number

$$a_2-a_1=a_3-a_2 \qquad a_3-a_2=a_4-a_3$$
$$a_4-a_3=a_5-a_4 \qquad a_5-a_4=a_6-a_5$$

**2.10. Ranking Function:**

Define a ranking function $R: f(R)\to \mathbb{R}$ which maps each fuzzy numbers to the real line. $f(R)$ represents the set of all hexagonal fuzzy numbers. If R be any linear ranking function, then

$$R(\tilde{A})=\left(\frac{a_1+a_2+a_3+a_4+a_5+a_6}{6}\right)$$

**2.11. Equivalent Hexagonal Fuzzy Numbers:**

A fuzzy number $\tilde{A}$ is said to be equivalent to a fuzzy number $\tilde{B}$ if its value of the ranking functions are the same. (ie) $\tilde{A}\approx\tilde{B}$ if $R(\tilde{A})=R(\tilde{B})$.

**2.12. Signed Distance Method:**

Defuzzification of $\tilde{A}$ can be found by signed distance method. If $\tilde{A}$ is a hexagonal fuzzy number, then the signed distance from $\tilde{A}$ to 0 is defined as

$$d\left(\tilde{A},0\right)=\frac{1}{2}\int_0^1\left(\left[A_L(\alpha),A_M(\alpha),A_R(\alpha)\right],\tilde{0}\right)d\alpha$$

, where $A_\alpha=\left[A_L(\alpha),A_M(\alpha),A_R(\alpha)\right]$

$$A_\alpha=\left[a+(b-a)\alpha,d-(d-c)\alpha,e+(e-f)\alpha\right],\alpha\in\left[0,1\right]$$

**2.13. Notations Used:**

A - Ordering cost per order
H - Holding cost per unit quantity per unit time
s - Shortage cost per unit quantity
S - Maximum order level
T - Length of the plan
D - Demand with time period [0,T]
Q - Order quantity per cycle
$T_C$ - Total cost for the period [0,T]
$T_{\tilde{C}}$ - Fuzzy total cost for the period [0,T]
F(Q) - De-fuzzified total cost for [0,T]
$F(Q)^*$ - Minimum de-fuzzified total cost for [0,T]
$Q_D$ - Optimal order quantity

**2.14. Assumptions:**

In this paper, the following assumptions are considered:
Total demand is considered as constant
Time of plan is constant
Shortage is allowed and it is completely backlogged.
Only holding cost, ordering cost and shortage cost are fuzzy in nature.

**2.15. Algorithm for finding fuzzy total cost and fuzzy optimal order quantity:**

**Step 1:** Calculate total cost for the crisp model for the given crisp values of A,H,S,T and D.
**Step 2:** Now, determine fuzzy total cost using fuzzy arithmetic operations on fuzzy holding cost, fuzzy ordering cost and fuzzy shortage cost, taken as trapezoidal fuzzy numbers.
**Step 3:** Use signed distance method for defuzzification. Then find fuzzy optimal order quantity which can be obtained by putting the first derivative of F(Q) equal to Zero and where the second derivative is positive at $Q=Q_D^*$

**III. FUZZY INVENTORY MODEL**

**3.1. Inventory model in crisp sense:**

First, an inventory model with shortages in crisp environment. The economic lot size is obtained by the following equation $Q=\sqrt{2DA}\sqrt{\frac{HT+S}{HTS}}$

Maximum order level is $S=\frac{Qs}{HT+s}$

The total cost for the period [0,T] is

$$Tc=\frac{HTs^2}{2Q}+\frac{s(Q-s)^2}{2Q}+\frac{AD}{Q}$$

$$Tc=\frac{HTQs^2}{2\left(HT+s\right)^2}+\frac{sQ\left(1-\frac{s}{HT+s}\right)^2}{2}+\frac{AD}{Q}$$

The optimum $Q^*$ and $Tc^*$ can be obtained by equating the first order partial derivatives of Tc to zero and solving the resulting equations**,**

Optimal order quantity.

$$Q^*=\sqrt{2DA}\sqrt{\frac{HT+S}{HTS}}$$

 

Minimum total cost

$$Q^{*} = \sqrt{2DA}\sqrt{\frac{HTS}{HT+S}}$$

**3.2. Inventory model in fuzzy sense:**

Consider the model in fuzzy environment. Since the ordering cost, holding cost and shortage cost are fuzzy in nature, represent them by Hexagonal fuzzy numbers. Let

$\tilde{A}$: Fuzzy ordering cost per order.

$\tilde{H}$: Fuzzy carrying or holding cost per unit quantity per unit time.

$\tilde{S}$: Fuzzy shortage cost per unit quantity per unit time.

The total demand and time of plan are considered as constants. Now fuzzifying the total cost. The fuzzy total cost is given by,

$$T\tilde{c} = \frac{\tilde{H}TQ\tilde{s}^2}{2(\tilde{H}T+\tilde{s})^2} + \frac{\tilde{s}Q\left(1-\frac{\tilde{s}}{\tilde{H}T+\tilde{s}}\right)^2}{2} + \frac{\tilde{A}D}{Q}$$

Our goal is to apply signed distance method to defuzzify the total cost and then obtain the optimal order quantity by using simple calculus techniques.

Suppose

$$\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$$

$$\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6)$$

$\tilde{S} = (s_1, s_2, s_3, s_4, s_5, s_6)$ are hexagonal fuzzy numbers. Then,

$$T\tilde{c} = \frac{(h_1,h_2,h_3,h_4,h_5,h_6)\otimes TQ \otimes (s_1,s_2,s_3,s_4,s_5,s_6)^2}{2\otimes\begin{pmatrix}(h_1,h_2,h_3,h_4,h_5,h_6)T\oplus \\ (s_1,s_2,s_3,s_4,s_5,s_6)\end{pmatrix}^2}$$

$$\oplus (s_1,s_2,s_3,s_4,s_5,s_6)\otimes Q \otimes$$

$$\left(1-\frac{\dfrac{(s_1,s_2,s_3,s_4,s_5,s_6)}{2\otimes((h_1,h_2,h_3,h_4,h_5,h_6)T\oplus s_1,s_2,s_3,s_4,s_5,s_6}}{2}\right)^2$$

$$\oplus \frac{(a_1,a_2,a_3,a_4,a_5,a_6)}{Q}\otimes D$$

$$T_{\tilde{C}} = \frac{h_1S_1^2TQ}{2(h_6T+S_6)^2} + \frac{S_1Q\left(1-\frac{S_6}{h_1T+S_1}\right)^2}{2} + \frac{a_1D}{Q},$$

$$\frac{h_2S_2^2TQ}{2(h_5T+S_5)^2} + \frac{S_2Q\left(1-\frac{S_5}{h_2T+S_2}\right)^2}{2} + \frac{a_2D}{Q},$$

$$\frac{h_3S_3^2TQ}{2(h_4T+S_4)^2} + \frac{S_3Q\left(1-\frac{S_4}{h_3T+S_3}\right)^2}{2} + \frac{a_3D}{Q},$$

$$\frac{h_4S_4^2TQ}{2(h_3T+S_3)^2} + \frac{S_4Q\left(1-\frac{S_3}{h_4T+S_4}\right)^2}{2} + \frac{a_4D}{Q},$$

$$\frac{h_5S_5^2TQ}{2(h_2T+S_2)^2} + \frac{S_5Q\left(1-\frac{S_2}{h_5T+S_5}\right)^2}{2} + \frac{a_5D}{Q},$$

$$\frac{h_6S_6^2TQ}{2(h_1T+S_1)^2} + \frac{S_6Q\left(1-\frac{S_1}{h_6T+S_6}\right)^2}{2} + \frac{a_6D}{Q}$$

$= (a,b,c,d,e,f)$ (say)

Now, $A_L(\alpha) = a + (b-a)\alpha$

$$= \frac{h_1S_1^2TQ}{2(h_6T+S_6)^2} + \frac{S_1Q\left(1-\frac{S_6}{h_1T+S_1}\right)^2}{2} + \frac{a_1D}{Q} +$$

$$\left\{\begin{array}{l}\left(\frac{h_2S_2^2}{(h_5T+S_5)^2} - \frac{h_1S_1^2}{(h_6T+S_6)^2}\right)\frac{TQ}{2} + \\ \left[S_2\left(1-\frac{S_5}{h_2T+S_2}\right)^2 - S_1\left(1-\frac{S_6}{h_1T+S_1}\right)^2\right]\frac{Q}{2} \\ +(a_2-a_1)\frac{D}{Q}\end{array}\right\}\alpha$$

And $A_M(\alpha) = d - (d-c)\alpha$

$$= \frac{h_4S_4^2TQ}{2(h_3T+S_3)^2} + \frac{S_4Q\left(1-\frac{S_3}{h_4T+S_4}\right)^2}{2} + \frac{a_4D}{Q} -$$

$$\left\{\begin{array}{l}\left(\dfrac{h_4S_4^2}{\left(h_3T+S_3\right)^2}-\dfrac{h_3S_3^2}{\left(h_4T+S_4\right)^2}\right)\dfrac{TQ}{2}+\\ \left[S_4\left(1-\dfrac{S_3}{h_4T+S_4}\right)^2-S_3\left(1-\dfrac{S_4}{h_3T+S_3}\right)^2\right]\dfrac{Q}{2}\\ +\left(a_4-a_3\right)\dfrac{D}{Q}\end{array}\right\}\alpha$$

And $A_R(\alpha)=e+(e-f)\alpha$

$$=\frac{h_6S_6^2TQ}{2\left(h_1T+S_1\right)^2}+\frac{S_6Q\left(1-\dfrac{S_1}{h_6T+S_6}\right)^2}{2}+\frac{a_6D}{Q}+$$

$$\left\{\begin{array}{l}\left(\dfrac{h_6S_6^2}{\left(h_1T+S_1\right)^2}-\dfrac{h_5S_5^2}{\left(h_2T+S_2\right)^2}\right)\dfrac{TQ}{2}+\\ \left[S_6\left(1-\dfrac{S_1}{h_6T+S_6}\right)^2-S_5\left(1-\dfrac{S_2}{h_5T+S_5}\right)^2\right]\dfrac{Q}{2}\\ +\left(a_6-a_5\right)\dfrac{D}{Q}\end{array}\right\}\alpha$$

Defuzzififying $T\tilde{c}$ by using signed distance method, we get,

$$d\left(T\tilde{c}(\tilde{A},\tilde{H},\tilde{S});0\right)=\frac{1}{2}\int_0^1\left[\begin{array}{l}A_L(\alpha)+A_M(\alpha)\\+A_R(\alpha)\end{array}\right]d\alpha$$

$$=\frac{1}{2}\int_0^1\left[\begin{array}{l}\dfrac{h_1S_1^2TQ}{2\left(h_6T+S_6\right)^2}+\dfrac{S_1Q\left(1-\dfrac{S_6}{h_1T+S_1}\right)^2}{2}+\dfrac{a_1D}{Q}+\\ \left\{\begin{array}{l}\left(\dfrac{h_2S_2^2}{\left(h_5T+S_5\right)^2}\dfrac{1}{2}-\dfrac{h_1S_1^2}{\left(h_6T+S_6\right)^2}\right)\dfrac{TQ}{2}+\\ \left[S_2\left(1-\dfrac{S_5}{h_2T+S_2}\right)^2-S_1\left(1-\dfrac{S_6}{h_1T+S_1}\right)^2\right]\dfrac{Q}{2}+\left(a_2-a_1\right)\dfrac{D}{Q}\end{array}\right\}\alpha\\ +\dfrac{h_4S_4^2TQ}{2\left(h_3T+S_3\right)^2}+\dfrac{S_4Q\left(1-\dfrac{S_3}{h_4T+S_4}\right)^2}{2}+\dfrac{a_4D}{Q}-\\ \left\{\begin{array}{l}\left(\dfrac{h_4S_4^2}{\left(h_3T+S_3\right)^2}-\dfrac{h_3S_3^2}{\left(h_4T+S_4\right)^2}\right)\dfrac{TQ}{2}+\\ \left[S_4\left(1-\dfrac{S_3}{h_4T+S_4}\right)^2-S_3\left(1-\dfrac{S_4}{h_3T+S_3}\right)^2\right]\dfrac{Q}{2}+\left(a_4-a_3\right)\dfrac{D}{Q}\end{array}\right\}\alpha\\ \dfrac{h_6S_6^2TQ}{2\left(h_1T+S_1\right)^2}+\dfrac{S_6Q\left(1-\dfrac{S_1}{h_6T+S_6}\right)^2}{2}+\dfrac{a_6D}{Q}+\\ \left\{\begin{array}{l}\left(\dfrac{h_6S_6^2}{\left(h_1T+S_1\right)^2}-\dfrac{h_5S_5^2}{\left(h_2T+S_2\right)^2}\right)\dfrac{TQ}{2}+\\ \left[S_6\left(1-\dfrac{S_1}{h_6T+S_6}\right)^2-S_5\left(1-\dfrac{S_2}{h_5T+S_5}\right)^2\right]\dfrac{Q}{2}+\left(a_6-a_5\right)\dfrac{D}{Q}\end{array}\right\}\alpha\end{array}\right]d\alpha$$

$$
=\frac{1}{2}\left[\begin{array}{l}
\dfrac{h_1S_1^2TQ}{2\left(h_6T+S_6\right)^2}+\dfrac{S_1Q\left(1-\dfrac{S_6}{h_6T+S_6}\right)^2}{2}+\dfrac{a_1D}{Q}-0 \\
+\left\{\begin{array}{l}\left(\dfrac{h_2S_2^2}{\left(h_5T+S_5\right)^2}-\dfrac{h_1S_1^2}{\left(h_6T+S_6\right)^2}\right)\dfrac{TQ}{4}+ \\
\left[S_2\left(1-\dfrac{S_5}{h_2T+S_2}\right)^2-S_1\left(1-\dfrac{S_6}{h_1T+S_1}\right)^2\right]\dfrac{Q}{4} \\
+\left(a_2-a_1\right)\dfrac{D}{2Q}-0\end{array}\right\} \\
+\dfrac{h_4S_4^2TQ}{2\left(h_3T+S_3\right)^2}+\dfrac{S_4Q\left(1-\dfrac{S_3}{h_4T+S_4}\right)^2}{2}+\dfrac{a_4D}{Q}-0 \\
\left\{\begin{array}{l}\left(\dfrac{h_4S_4^2}{\left(h_3T+S_3\right)^2}-\dfrac{h_3S_3^2}{\left(h_4T+S_4\right)^2}\right)\dfrac{TQ}{4}+ \\
\left[S_4\left(1-\dfrac{S_3}{h_4T+S_4}\right)^2-S_3\left(1-\dfrac{S_4}{h_3T+S_3}\right)^2\right]\dfrac{Q}{4} \\
+\left(a_4-a_3\right)\dfrac{D}{2Q}-0\end{array}\right\} \\
\dfrac{h_6S_6^2TQ}{2\left(h_1T+S_1\right)^2}+\dfrac{S_6Q\left(1-\dfrac{S_1}{h_6T+S_6}\right)^2}{2}+\dfrac{a_6D}{Q}-0 \\
\left\{\begin{array}{l}\left(\dfrac{h_6S_6^2}{\left(h_1T+S_1\right)^2}-\dfrac{h_5S_5^2}{\left(h_2T+S_2\right)^2}\right)\dfrac{TQ}{4}+ \\
\left[S_6\left(1-\dfrac{S_1}{h_6T+S_6}\right)^2-S_5\left(1-\dfrac{S_2}{h_5T+S_5}\right)^2\right]\dfrac{Q}{4} \\
+\left(a_6-a_5\right)\dfrac{D}{2Q}-0\end{array}\right\}
\end{array}\right]\frac{\alpha}{2}
$$

$$
=\frac{D}{6Q}\left[a_1+a_2+a_3+a_4+a_5+a_6\right]+
$$

$$
\frac{TQ}{12}\left[\begin{array}{l}\dfrac{h_1s_1{}^2}{\left[h_6T+s_6\right]^2}+\dfrac{h_2s_2{}^2}{\left[h_5T+s_5\right]^2}+\dfrac{h_3s_3{}^2}{\left[h_4T+s_4\right]^2} \\
+\dfrac{h_4s_4{}^2}{\left[h_3T+s_3\right]^2}+\dfrac{h_5s_5{}^2}{\left[h_2T+s_2\right]^2}+\dfrac{h_6s_6{}^2}{\left[h_1T+s_1\right]^2}\end{array}\right]
$$

$$
+\frac{Q}{12}\left[\begin{array}{l}S_1\left(1-\dfrac{S_6}{h_1T+S_1}\right)^2+S_2\left(1-\dfrac{S_5}{h_2T+S_2}\right)^2+ \\
S_3\left(1-\dfrac{S_4}{h_3T+S_3}\right)^2+S_4\left(1-\dfrac{S_3}{h_4T+S_4}\right)^2+ \\
S_5\left(1-\dfrac{S_2}{h_5T+S_5}\right)^2+S_6\left(1-\dfrac{S_1}{h_6T+S_6}\right)^2\end{array}\right]
$$

Computation of $Q_D^*$ at which F(Q) is minimum.

F(Q) is minimum when $\dfrac{dF(Q)}{dQ}=0$ and where

d $\alpha$ , $\dfrac{d^2F(Q)}{dQ^2}>0$.

$$
Q_D^*=\sqrt{\frac{2D[a_1+a_2+a_3+a_4+a_5+a_6]}{\begin{array}{l}T\left[\begin{array}{l}\dfrac{h_6T+S_6{}^2}{\left(h_6T+S_6\right)^2}+\dfrac{h_5T+S_5{}^2}{\left(h_5T+S_5\right)^2}+\dfrac{h_4T+S_4{}^2}{\left(h_4T+S_4\right)^2}+ \\
\dfrac{h_3T+S_3{}^2}{\left(h_3T+S_3\right)^2}+\dfrac{h_2T+S_2{}^2}{\left(h_2T+S_2\right)^2}+\dfrac{h_1T+S_1{}^2}{\left(h_1T+S_1\right)^2}\end{array}\right] \\
+s_1\left(1-\dfrac{s_6}{h_1T+s_1}\right)^2+s_2\left(1-\dfrac{s_5}{h_2T+s_2}\right)^2+ \\
s_3\left(1-\dfrac{s_4}{h_3T+s_3}\right)^2+s_4\left(1-\dfrac{s_3}{h_4T+s_4}\right)^2+ \\
s_5\left(1-\dfrac{s_2}{h_5T+s_5}\right)^2+s_6\left(1-\dfrac{s_1}{h_6T+s_6}\right)^2\end{array}}}
$$

## IV.NUMERICAL EXAMPLE

Let D=500 unit T=6 Days

$\tilde{A}$ =(15,17,18,22,23,25)

$\tilde{H}$ =(7,9,10,14,15,17)

$\tilde{S}$ = (2,3,4,8,9,10)

### 4.1. Crisp Model:

$$Q^*=\sqrt{2DA}\sqrt{\frac{HT+S}{HTS}}$$

$$=\sqrt{2\times500\times20}\sqrt{\frac{12\times6+6}{12\times6\times6}}$$

$Q^*$ **= 60.08**

**Minimum Total Cost:**

$$T_C^* \quad =\sqrt{2DA}\sqrt{\frac{HTS}{HT+S}}$$

$$= \sqrt{2\times 500\times 20}\sqrt{\frac{12\times 6\times 6}{12\times 6+6}}$$

**=332.79**

**4.2. Fuzzy Model:**

$$Q_D^* = \sqrt{\frac{2D[a_1+a_2+a_3+a_4+a_5+a_6]}{T\left[\begin{array}{l}\frac{h_6T+S_6^2}{(h_6T+S_6)^2}+\frac{h_5T+S_5^2}{(h_5T+S_5)^2}+\frac{h_4T+S_4^2}{(h_4T+S_4)^2}\\+\frac{h_3T+S_3^2}{(h_3T+S_3)^2}+\frac{h_2T+S_2^2}{(h_2T+S_2)^2}+\frac{h_1T+S_1^2}{(h_1T+S_1)^2}\end{array}\right]} \begin{array}{l}+s_1\left(1-\frac{s_6}{h_1T+s_1}\right)^2+s_2\left(1-\frac{s_5}{h_2T+s_2}\right)^2+\\ s_3\left(1-\frac{s_4}{h_3T+s_3}\right)^2+s_4\left(1-\frac{s_3}{h_4T+s_4}\right)^2+\\ s_5\left(1-\frac{s_2}{h_5T+s_5}\right)^2+s_6\left(1-\frac{s_1}{h_6T+s_6}\right)^2\end{array}}$$

**= 61.58**

$$F(Q^*) = \frac{D}{6Q_{D^*}}\left[a_1+a_2+a_3+a_4+a_5+a_6\right]$$

$$+\frac{TQ_D^*}{12}\left[\begin{array}{l}\frac{h_1s_1^2}{(h_6T+s_6)^2}+\frac{h_2s_2^2}{(h_5T+s_5)^2}+\frac{h_3s_3^2}{(h_4T+s_4)^2}+\\ \frac{h_4s_4^2}{(h_3T+s_3)^2}+\frac{h_5s_5^2}{(h_2T+s_2)^2}+\frac{h_6s_6^2}{(h_1T+s_1)^2}\end{array}\right]$$

$$+\frac{Q_D^*}{12}\left[\begin{array}{l}s_1\left(1-\frac{s_6}{(h_1T+s_1)}\right)^2+s_2\left(1-\frac{s_5}{(h_2T+s_2)}\right)^2+\\ s_3\left(1-\frac{s_4}{(h_3T+s_3)}\right)^2+s_4\left(1-\frac{s_3}{(h_4T+s_4)}\right)^2+\\ s_5\left(1-\frac{s_2}{(h_5T+s_5)}\right)^2+s_6\left(1-\frac{s_1}{(h_6T+s_6)}\right)^2\end{array}\right]$$

$$=162.3904+19.4162+142.9409$$

$$F(Q^*)=324.7475$$

Then, $Q^*$ = 61.58 units and $F(Q^*)$ = Rs. 324.7475

## V. CONCLUSION

In this paper, fuzzy optimal order quantity and fuzzy optimal total cost is studied with aid of hexagonal fuzzy number. To estimate various fuzzy optimal quantities, holding cost, ordering cost and shortage cost using hexagonal fuzzy numbers have been utilized. New arithmetical operations of hexagonal fuzzy numbers are proposed to get the expected result. Hence, fuzzy values are closer to the crisp values of the system.

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