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Prime – Antimagic Labeling of Some Special Classes Of Graphs

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Abstract

An Prime Antimagic Labeling of a finite simple undirected graph with p vertices and q edges is a bijection from the set of vertices to the integers { 1, 2, ... p } such that for each edge uv , the labels assigned to u and v are relatively prime and the induced edge labeling f (uv) = f (u) + f (v) are all distinct. A Graph is called Prime Antimagic if it admits Prime Antimagic Labeling. In this article we study the new classes of graphs $K_n^c + K_2$, $P_{n_{(m)}}$, SF(n,m) and Fans are Prime Antimagic graph. We also discuss Prime Odd Antimagic labeling of theta graph , fan graph and in the contex of graph operation fusion.

Key Words: Prime Antimagic Labeling, Prime Odd Antimgic labeling, $K_n^c + K_2$, $P_{n_{(m)}}$, SF(n,m), Fans, theta graph and in the contex of graph operation fusion

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1. Introduction

All graphs in this paper are finite, simple, undirected and without loops unless otherwise stated. In 1990, Hartsfield and Ringel [3] introduced the concepts called antimagic labeling and antimagic graph. We follow the notation and terminology of [7].

Definition 1.1

For a graph G = (V, E) with p vertices and q edges and without any isolated vertex, an *antimagic vertex labeling* is a bijection f : V → { 1, 2, ... p } such that the induced edge sum f* : E → N given by f (uv) = f (u) + f (v) , u, v ∈ V is injective. A graph is called *antimagic* if it admits an antimagic labeling.

The notion of a prime labeling was originated by Roger Entringer and was discussed in a paper by Tout [5].

Definition 1.2

A *Prime Labeling* of a graph G is an injective function f : V → { 1, 2, ... p } such that for every pair of adjacent vertices u and v, gcd (f(u), f(v)) = 1. The graph which admits prime labeling is called a prime graph.

We will give brief summary of definitions [1], [2], [4] and [6] which are useful for our present investigation.

Definition 1.3

An *SF (n,m)* is a graph consisting of a cycle C_n , n ≥ 3 , and n set of m independent vertices where each set joins each of the vertices of C_n.

Definition 1.4

The graph $P_{n_{(m)}}$ = G (V , E) such that V (G) = { v_{ij} , 1 ≤ i ≤ m, and 1 ≤ j ≤ n }

E (G) = { $v_{ij}v_{i(j+1)}$ / 1 ≤ i ≤ m and 1 ≤ j ≤ n – 1 } ∪ { $v_{i(n-1)}v_{i+1((n-1))}$ / 1 ≤ i ≤ m – 1 }

Definition 1.5

A *theta graph* is a block with two non – adjacent vetices of degree 3 and all other vertices of degree 2 is called a theta graph.

Definition 1.6

Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by *fusing (identifying)* two vertices u and v by a single vertex x in G_1 such that every edge which has incident with either u or v in G now incident with x in G_1.

Definition 1.7

Bi-Star is the graph obtained by joining the apex vertices of two copies of Star$K_{1,n}$.

Definition 1.8

The *friendship graph* F_n is one – point union of n copies of cycle C_3

2. Main Results

Definition 2.1

A ***Prime - Antimagic labeling*** of a graph G is an bijective function f : V(G) → {1,2,…|V|} such that every pair of adjacent vertices u and v, g.c.d (f(u), f(v)) = 1 and the induced mapping f* : E(G) → N defined by f* (e = *uv*) = $\sum f(u, v)$ where (u,v) ∈ E(G) is injective and all these edge labelings are distinct.

Theorem 2.2

The graph SF (n, 1) admits a Prime Antimagic labeling

Proof:

Let G denote the graph SF (n, 1).

Let v_1, v_2, ... v_n be the vertices of the cycle SF(n,1) and v_j' for j = 1,2,...n be the vertices joining the corresponding vertices v_j.

Here | V (G) | = 2n and | E (G) | = 2n.

Define f: V → { 1,2,...| V | } by

f (v_j) = 2j – 1 for j = 1,2,..n – 1

f (v_j) = 2n for j = n

f (v_j') = 2j , j = 1,2,...n – 1

f (v_j') = 2j – 1 for j = n.

Therefore, there exists a bijection f : V → { 1,2,...| V | } such that each vertex u and v satisfies gcd(f(u), f(v)) = 1.

The distinct edge labels are determined by the condition e($v_j v_{j+1}$) = f(v_j) + f(v_{j+1}).

Hence Proved.

Theorem 2.3

K_n^c + K_2 is Prime Antimagic labeling, if n ≤ 5 and n is odd.

Proof:

Let G = K_n^c + K_2.

Let u and w be the vertices of K_2 andv_1,v_2, ... v_n be the vertices of K_n^c.

Let V (G) = {u,w, v_i / 1 ≤ i ≤ n }

E (G) = {uw, uv_i , wv_i / 1 ≤ i ≤ n}.

Therfore | V (G) | = n + 2 and | E (G) | = 2n + 1.The vertex labelings are defined by f : V(G) → { 1,2,... n + 2 }

by f(u) = 1, f (w) = n + 2 , f(v_i) = i + 1, 1 ≤ i ≤ n.

The edge labels f*: E (G) → { 3,4,... 2n – 1 } are distinct.

Thus proved.

Theorem 2.4

The graph $P_{n_{(m)}}$ is a Prime Antimagic graph for all n, m ≥ 2.

Proof:

Let G = $P_{n_{(m)}}$.

Let V (G) = { v_{ij} , 1 ≤ i ≤ m, and 1 ≤ j ≤ n }

E (G) = { $v_{ij}v_{i(j+1)}$ / 1 ≤ i ≤ m and 1 ≤ j ≤ n – 1 } ∪ { $v_{i(n-1)}v_{i+1((n-1)}$ / 1 ≤ i ≤ m – 1 }

Then | V (G) | = mn and | E(G) | = mn – 1.

Define f: V → {1, 2...mn } by f (v_{ij}) = n (i – 1) + j ,

1 ≤ i ≤ m, and 1 ≤ j ≤ n and the prime condition is

gcd (f(v_{ij}) , f($v_{i(j+1)}$)) = 1. All the edge labels are distinct.

Hence the theorem follows.

Theorem 2.5

The Bi- Star $B_{n,n}$ admits prime Antimagic Labeling.

Proof:

Consider the two copies of $K_{1,n}$. Let v_1, v_2, ... v_n and u_1, u_2, ... u_n be the corresponding vertices of each copy of $K_{1,n}$ with apex vertex v and u.

Let $e_i = v\, v_i$, $e_i' = u\, u_i$ and $e = uv$ of bistar graph.

Here | V ($B_{n,n}$) | = 2n + 2, | E ($B_{n,n}$) | = 2n + 1.

Define a prime labeling f: V (G) → {1, 2,…|V|} as follows.

Case (i) : n is odd

f(u) = 1

f(v) = n

f(u_i) = 2i, 1 ≤ i ≤ n

Case (ii) : n is even

f(u) = 1

f(v) = 7

f(u_i) = 2i – 2 , 1 ≤ i ≤ n

f(v_i) = 2i – 1 ≠ n and 1 ≤ i ≤ n

f(v_n) = 2n + 2

f(v_1) = 3

f(v_2) = 5, f(v_3) = 9 and so on.

The induced edge labels are distinct and the theorem follows.

3. Prime – Odd Antimagic labeling

In this section we introduce the another new concept of Prime – odd Antimagic labeling and Prime – odd Antimagic labeling of the theta graph , fusion of the graph..

Definition 3.1

A connected graph G with |V| = p vertices and |E| = q edges is said to be ***Prime – odd Antimagic labeling*** if there is an bijection f : V(G) →{1,3,….2 |V| - 1} such that every pair of adjacent vertices u and v, g.c.d(f (u),f (v)) = 1 and the induced mapping f* : E(G) → N defined by f*(e = uv) =$\sum f(u,v)$ where (u,v) ∈ E(G) is injective and all these edge labelings are distinct.

Theorem 3.2

The Theta graph T_a admits a prime odd antimagic labeling.

Proof:

Let T_a be the theta graph with centre v_4, V(T_a) = { v_1, v_2, ... v_7 } and

E (T_a) = { v_iv_{i+1} / 1 ≤ i ≤ 6 } ∪{ v_1v_5 }∪{ v_3v_7}
Define a label f: V (T_a) → { 1,3,...2p - 1 } such that gcd (f(v_i) , f(v_j)) = 1 and for the vertex v_1, v_7, v_3 gcd $\left(f(v_1), f(v_5)\right) = 1$ and
gcd (f(v_7) , f(v_3)) = 1 and defined by
f (v_i) = 2i – 1 , 1 ≤ i ≤ 7. The edge labels f* (e) = $\sum f(u)$ are distinct.
Hence T_a is an prime odd antimagic graph.

Observation 3.3
The friendship graph F_n admits a prime odd antimagic labeling, if n ≤ 3.

Theorem 3.3

The fusion of any two vertices in the cycle of T_a is an prime odd antimagic graph.

Proof:

Let T_a be the theta graph with centre v_4, V(T_a) = { v_1, v_2, ... v_7 } and E(T_a) = { $v_i v_{i+1}$ / 1 ≤ i ≤ 6 } ∪{ $v_1 v_5$ }∪{ $v_3 v_7$ }.

Then | V (T_a) | = 7 and | E(T_a) | = 8.

Let G be a graph obtained by fusion of two vertices $v_i v_{i+1}$ in the cycle of T_a.

Thus | V (G) | = 6 and | E (G) | = 7.

The vertices of a new graph G is denoted by u_1,u_2, u_3, u_4 , u_5, u_6 and u_6 is the centre vertex of G.

Define a label f: V (G) → {1, 3, 5, 7 9, 11} such that

f (u_i) = 2i – 1 , i = 1, 2,...6.

Here $v_1 v_2$ are the fused by a single vertexu_1.

That is $v_1 = v_3$ = u.

For each vertex gcd $\left(f(u_i), f(u_j)\right) = 1$ and each edge

e = $u_i u_j$ ∈ G, f* (e) = $\sum f(u)$ are all distinct.

Hence proved.

Theorem 3.4

The graph obtained by identifying any two vertices v_i and v_j where d (v_i, v_j) ≥ 3 of cycle C_n is an prime odd antimagic labeling.

Proof:

Let C_n be the cycle with vertices v_1, v_2, ... v_n and the vertex v_1 fused with v_m where m ≤ n/2 .

Denote the resultant graph as G. Here we note that | V (G) | = n – 1. It is obvious that identifying two vertices of cycle C_n produces connected graph which includes two edge disjoint cycles C_{m-1} and C_{n-m-1}.

To define labeling f: V(G) →{1,3,….2 |V| - 1} by

f (v_i) = 2i – 1, i = 1,2,..n and all edge labelings are distinct. Hence proved.

3. Conclusion

We have presented the prime antimagic labeling of certain classes of graphs such as $K_n^c + K_2$, $P_{n(m)}$and SF(n,m). We also discuss Prime Odd Antimagic labeling of theta graph , fan graph and in the contex of graph operation fusion.

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