A New Method for Forecasting Pulses Productivity Data Based on Fuzzy Time Series with Higher Forecast Accuracy Rate

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Abstract **-The time-series models have been used to make predictions in whether forecasting, agriculture forecasting, academic enrollments, etc. Reference [7] introduced the concept the concept of fuzzy time series in 1993. Over the past 20 years, many fuzzy time series methods have been proposed for forecasting enrollments. These methods mainly focus on three factors, namely, the universe of discourse, partition of discourse and the defuzzification method. These methods have either used pulses productivity data or difference of productivity data or percentage change as the universe of discourse. And either used frequency density based portioning or ratio-based portioning as the partition of discourse. The main issue in forecasting is in improving accuracy. But the forecasting accuracy rate of the existing methods is not good enough. This paper proposed a method based on fuzzy time series, which gives the higher forecasting accuracy rate than the existing methods. The proposed method used the percentage change as the universe of discourse; mean based partitioning as the partition of discourse and proposed method for defuzzification. To illustrate the forecasting process, the pulses productivity data of Tamil Nadu is used.**

Keywords- **Defuzzification, Fuzzification, Fuzzy time series, Fuzzy set, Hao-Tien Liu method, Heuristic time-invariant method.**

I. INTRODUCTION

Forecasting plays an important role in our day-to-day life. Many different approaches such as Linear, Non-Linear, Auto Regressive Integrated Moving Average models have been proposed in literature since years for the analyses of area, production and productivity of major crops. In recent years, nonparametric regression technique for functional estimation has become an increasingly popular tool for data analysis. The increased data availability and explosion of computing power has now made it possible to use a wide range of modern nonparametric regression techniques in time-series data. Fuzzy time series forecasting models do not require assumptions that stochastic models do. On the other hand, most of the time series encountered in real life should be considered as fuzzy time series due to the uncertainty that they contain and they should be analyzed with models appropriate to fuzzy set theory. Reference [7]-[9] used the fuzzy set theory given by [13] & [14] to develop the time-variant and time-invariant models for fuzzy time series forecasting, and considered it on a problem of forecasting student enrollments using time series data of the students at the University of Alabama. Reference [2] presented a simplified method of fuzzy time series forecasting of enrollments using arithmetic operations. The Fuzzy time series method involves three steps which are Fuzzification, Identification of the fuzzified relations and Defuzzification respectively. Many studies on these three steps have been done in literature because these steps have either positive or negative impact on the forecasting performance of a method. While some of the approaches proposed in the literature involve first-order forecasting models, whereas some involve higher order forecasting models. Reference [2] , [7]-[9] and [12] can be given as examples of first-order fuzzy time series forecasting models whereas, [1] and [3] studies involve high-order fuzzy time series forecasting models. Further, many researchers, [1], [6], [4]-[5], and [11] worked on the development of various models of fuzzy time series forecasting and its implementation.

The aim of this paper is to propose a method that is aimed to attain better forecasting accuracy by using fuzzy time series. For forecast only time series data will be used in numerical form without any additional pieces of knowledge. In this paper, a new method is proposed for forecasting pulses productivity data of Tamil Nadu based on fuzzy time series with higher forecast accuracy rate. The proposed method used the percentage change as the universe of discourse; mean based partitioning as the partition of discourse and proposed method for defuzzification. In section 2, the basic concepts and definitions of fuzzy time series are given. In section 3, the proposed method and the comparison of the proposed method with the existing methods is given. Finally the concluding remarks are discussed in section 4.

II. CONCEPTS OF FUZZY TIME SERIES

A. Definition Of Fuzzy Time Series

Fuzzy time series is assumed to be a fuzzy variable along with associated membership function. Song and Chissom [1993] have proposed a procedure for solving fuzzy time series model described in the following steps. Let U be the universe of discourse, where $U = \{u_1, u_2, . . . , u_n\}$. A fuzzy set A_i of U is defined by,

$$A_i = \mu_{Ai}(\mu_1)/u_1 + \mu_{Ai}(\mu_2)/u_2 + . . . + \mu_{Ai}(\mu_n)/u_n \quad (2.1)$$

where μ_{Ai} is the membership function of fuzzy set A_i, $\mu_{Ai} : U \rightarrow [0, 1]$

$\mu_{Ai}(U_i)$ denotes the membership value of U_i in A_i, $\mu_{Ai}(\mu_i) \in [0, 1]$ and $1 \le i \le n$.

Song and Chissom (1993a, b, 1994) presented the following definitions of the fuzzy time series.

Definition 1: Fuzzy Time Series

Let $Y(t), (t = 0, 1, 2, ...)$, is a subset of real number R. Let $Y(t)$ be the universe of discourse defined by the fuzzy set $\mu_i(t)$. If $F(t)$ consists of $\mu_i(t)$ (i = 1, 2,…). *F(t)* is called a fuzzy time series on Y(t). In definition 1, *F(t)* can be viewed as a linguistic variables. This represents for the main difference between fuzzy time series and classical time series, whose values must be real numbers.

Definition 2: Time – Invariant Fuzzy Time Series

Suppose $F(t)$ is caused only by $F(t-1)$ and is denoted by $F(t-1) \rightarrow F(t)$; if there exists a fuzzy relationship between $F(t)$ and $F(t-1)$ can be expressed as the fuzzy relational equation

$$F(t) = F(t-1) \circ R(t, t-1)$$

Here ‘‘∘’’ is max–min composition operator. The relation R is called first-order model of $F(t)$. Further, if fuzzy relation $R(t, t-1)$ of $F(t)$ is independent of time t, that is to say for different times t_1 and t_2, $R(t_1, t_1-1) = R(t_2, t_2-1)$, then $F(t)$ is called a time - invariant fuzzy time series. Otherwise is called a time – variant fuzzy time series.

B. Performance Of Models

The following measures of goodness of fit have been used to study the performances of different models.

Root Mean Square Error (RMSE) = $\left[\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 / n\right]^{1/2}$

Mean Absolute Error (MAE) = $\sum_{i=1}^{n}\left|(Y_i - \hat{Y}_i\right| / n$, and

Mean Square Error (MSE) = $\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 / (n-p)$

Average Forecasting Error Rate (AFER) = $\dfrac{\left|Y_i - \hat{Y}_i\right| / Y_i}{n} \times 100\%$

where n and p are number of observations and number of parameters, respectively in the model. The lower the values of these statistics, the better are the fitted model.

III. PROPOSED METHOD

In this section, a new method has been proposed for forecasting total pulses productivity of Tamil Nadu based on fuzzy time series with higher forecast accuracy rate. This proposed method uses the percentage change as the universe of discourse; mean based partitioning as the partition of discourse and again the same method for defuzzification. The proposed method is based on the following model, which consists of six phases, as shown in Fig. 1. The year to year percentage change of total pulses productivity data of the Tamil Nadu is given in Table I.

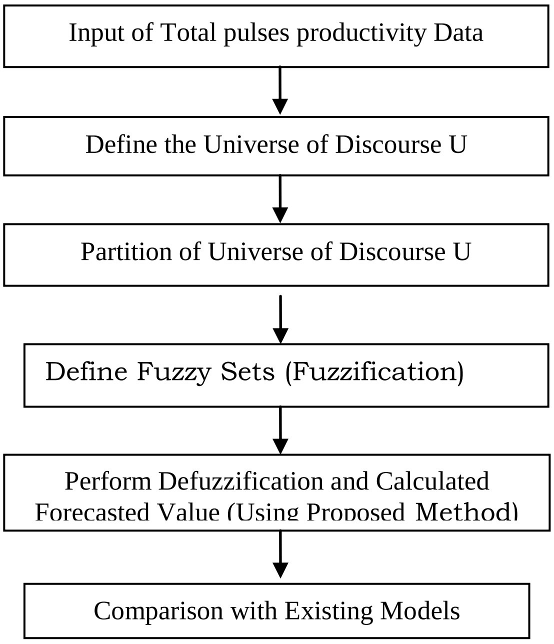

Fig.1 Procedure of the Proposed Method

Step 1: Define the universe of discourse U and partition it into intervals of equal length. The percentage change of pulses productivity data from year to year is given in Table I and ranges from -43.99% to 60.53%. For example, assume that the universe of discourse U = [-44, 61] is partitioned into fifty-three equal intervals.

TABLE I THE YEAR TO YEAR PERCENTAGE CHANGE OF PULSES PRODUCTIVITY DATA

Step 2: Get a mean of the original data. Get the means of frequency of each interval shown in Table II. Compare the means of original and frequency of each interval and then split the fifty three intervals into number of sub-intervals respectively.

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TABLE II MEANS OF ORIGINAL DATA AND FREQUENCY OF INTERVALS DATA

Interval	Number of Data	Interval	Number of Data
[-44.0, -42.0]	1	[10.0, 12.0]	0
[-42.0, -40.0]	0	[12.0, 14.0]	1
[-40.0, -38.0]	0	[14.0, 16.0]	1
[-38.0, -36.0]	0	[16.0, 18.0]	2
[-36.0, -34.0]	0	[18.0, 20.0]	1
[-34.0, -32.0]	0	[20.0, 22.0]	0
[-32.0, -30.0]	0	[22.0, 24.0]	3
[-30.0, -28.0]	0	[24.0, 26.0]	0
[-28.0, -26.0]	0	[26.0, 28.0]	0
[-26.0, -24.0]	0	[28.0, 30.0]	0
[-24.0, -22.0]	0	[30.0, 32.0]	0
[-22.0, -20.0]	1	[32.0, 34.0]	0
[-20.0, -18.0]	0	[34.0, 36.0]	0
[-18.0, -16.0]	1	[36.0, 38.0]	0
[-16.0, -14.0]	0	[38.0, 40.0]	0
[-14.0, -12.0]	3	[40.0, 42.0]	0
[-12.0, -10.0]	1	[42.0, 44.0]	0
[-10.0, -8.0]	4	[44.0, 46.0]	0
[-8.0, -6.0]	0	[46.0, 48.0]	0
[-6.0, -4.0]	3	[48.0, 50.0]	0
[-4.0, -2.0]	2	[50.0, 52.0]	0
[-2.0, 0.0]	7	[52.0, 54.0]	0
[0.0, 2.0]	6	[54.0, 56.0]	0
[2.0, 4.0]	7	[56.0, 58.0]	0
[4.0, 6.0]	7	[58.0, 60.0]	0
[6.0, 8.0]	3	[60.0, 62.0]	1
[8.0, 10.0]	4	-	-

Step 3: Define each fuzzy set X_i based on the redivided intervals and fuzzify the Total productivity data shown in Table III, where fuzzy set X_i, denotes a linguistic value of the year to year percentage change represented by fuzzy set.

TABLE III FUZZY INTERVALS USING MEAN BASED PARTITIONING

Step 4: Defuzzify the fuzzy data shown in Table IV, using the Proposed Method. Comparison of different forecasting methods are shown in Table IV. In Fig. 2 shows the trends in pulses productivity based on existing and proposed method. The characteristics of model performance parameters like RMSE, MAE,MSE and AFER are shown in Table VI.

TABLE IV FORECASTING RESULTS OF THE PROPOSED METHOD

TABLE V COMPARISON OF DIFFERENT FORECASTING METHODS

Fig. 2 Trends in Pulses productivity based on Hao-Tien Liu, Heuristic Time-Invariant fuzzy time-series and Proposed Method

TABLE VI CHARACTERISTICS OF MODEL PERFORMANCE PARAMETERS

IV. CONCLUSIONS

The study demonstrated that the proposed model is justified to be a suitable model to study the trends in productivity of total pulses crop since it shows a low value of RMSE, MAE, MSE and AFER which is desirable. Also a Decreasing trends in productivity of total pulses crops have been observed. Based on this fact, we can conclude that the proposed method achieves a better result.

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TABLE I THE YEAR TO YEAR PERCENTAGE CHANGE OF PULSES PRODUCTIVITY DATA

Year to Year	Change	Year to Year	Change	Year to Year	Change
1950-51	7.47	1970-71	23.62	1990-91	6.59
1951-52	9.63	1971-72	-9.55	1991-92	2.43
1952-53	17.56	1972-73	1.98	1992-93	-13.58
1953-54	4.56	1973-74	-20.06	1993-94	22.69
1954-55	2.78	1974-75	9.31	1994-95	-17.89
1955-56	-1.93	1975-76	20.00	1995-96	-0.99
1956-57	2.36	1976-77	4.01	1996-97	3.25
1957-58	0.00	1977-78	5.34	1997-98	15.74
1958-59	1.15	1978-79	-9.30	1998-99	-12.13
1959-60	0.76	1979-80	0.62	1999-2000	8.10
1960-61	-1.89	1980-81	4.01	2000-01	-13.00
1961-62	3.08	1981-82	13.65	2001-02	-9.87
1962-63	-2.24	1982-83	-4.18	2002-03	5.34
1963-64	-0.38	1983-84	9.81	2003-04	-2.13
1964-65	-1.92	1984-85	17.37	2004-05	-8.17
1965-66	1.17	1985-86	-4.23	2005-06	60.53
1966-67	-1.54	1986-87	-1.55	2006-07	-43.99
1967-68	-5.49	1987-88	-10.99	2007-08	2.97
1968-69	6.64	1988-89	2.52	2008-09	22.12
1969-70	5.45	1989-90	4.42	-	-

TABLE III FUZZY INTERVALS USING MEAN BASED PARTITIONING

Linguistic	Interval	Linguistic	Interval
X_1	[-44.00, -42.00]	X_{31}	[2.29, 2.58]
X_2	[-22.00, -20.00]	X_{32}	[2.58, 2.87]
X_3	[-18.00, -16.00]	X_{33}	[2.87, 3.16]
X_4	[-14.00, -13.33]	X_{34}	[3.16, 3.45]
X_5	[-13.33, -12.66]	X_{35}	[3.45, 3.74]
X_6	[-12.66, -12.00]	X_{36}	[3.74, 4.00]
X_7	[-12.00, -10.00]	X_{37}	[4.00, 4.29]
X_8	[-10.00, -9.50]	X_{38}	[4.29, 4.58]
X_9	[-9.50, -9.00]	X_{39}	[4.58, 4.87]
X_{10}	[-9.00, -8.50]	X_{40}	[4.87, 5.16]
X_{11}	[-8.50, -8.00]	X_{41}	[5.16, 5.45]
X_{12}	[-6.00, -5.33]	X_{42}	[5.45, 5.74]
X_{13}	[-5.33, -4.66]	X_{43}	[5.74, 6.00]
X_{14}	[-4.66, -4.00]	X_{44}	[6.00, 6.67]
X_{15}	[-4.00, -3.00]	X_{45}	[6.67, 7.34]
X_{16}	[-3.00, -2.00]	X_{46}	[7.34, 8.00]
X_{17}	[-2.00, -1.71]	X_{47}	[8.00, 8.50]
X_{18}	[-1.71, -1.42]	X_{48}	[8.50, 9.00]
X_{19}	[-1.42, -1.13]	X_{49}	[9.00, 9.50]
X_{20}	[-1.13, -0.84]	X_{50}	[9.50, 10.00]
X_{21}	[-0.84, -0.55]	X_{51}	[12.00, 14.00]
X_{22}	[-0.55, -0.26]	X_{52}	[14.00, 16.00]
X_{23}	[-0.26, 0.00]	X_{53}	[16.00. 17.00]
X_{24}	[0.00, 0.33]	X_{54}	[17.00, 18.00]
X_{25}	[0.33, 0.66]	X_{55}	[18.00, 20.00]
X_{26}	[0.66, 0.99]	X_{56}	[22.00, 22.67]
X_{27}	[0.99, 1.32]	X_{57}	[22.67, 23.34]
X_{28}	[1.32, 1.65]	X_{58}	[23.34, 24.00]
X_{29}	[1.65, 2.00]	X_{59}	[60.00, 62.00]
X_{30}	[2.00, 2.29]	-	-

TABLE IV FORECASTING RESULTS OF THE PROPOSED METHOD

Year	Productivity	Percentage	Fuzzy set	FIM	Forecast	PP	FER
1950	174	-	-	-	-	-	-
1951	187	7.47	X_{46}	7.67	187.35	7.67	0.0031
1952	205	9.63	X_{50}	9.75	205.23	9.75	0.0019
1953	241	17.56	X_{54}	17.50	240.88	17.50	0.0008
1954	252	4.56	X_{38}	4.44	251.66	4.42	0.0023
1955	259	2.78	X_{32}	2.73	258.88	2.73	0.0008
1956	254	-1.93	X_{17}	-1.86	254.18	-1.86	0.0012
1957	260	2.36	X_{31}	2.44	260.20	2.44	0.0013
1958	260	0.00	X_{24}	0.17	260.44	0.17	0.0028
1959	263	1.15	X_{27}	1.16	263.02	1.16	0.0001
1960	265	0.76	X_{26}	0.83	265.18	0.83	0.0011
1961	260	-1.89	X_{17}	-1.86	260.07	-1.86	0.0004
1962	268	3.08	X_{33}	3.02	267.85	3.02	0.0009
1963	262	-2.24	X_{16}	-2.50	261.30	-2.50	0.0045
1964	261	-0.38	X_{22}	-0.41	260.93	-0.41	0.0004
1965	256	-1.92	X_{17}	-1.86	256.15	-1.86	0.0010
1966	259	1.17	X_{27}	1.16	258.97	1.16	0.0002
1967	255	-1.54	X_{18}	-1.57	254.93	-1.57	0.0005
1968	241	-5.49	X_{12}	-5.67	240.54	-5.67	0.0032
1969	257	6.64	X_{44}	6.34	256.28	6.34	0.0047
1970	271	5.45	X_{42}	5.60	271.39	5.60	0.0024
1971	335	23.62	X_{58}	23.67	335.15	23.67	0.0007
1972	303	-9.55	X_{8}	-9.75	302.34	-9.75	0.0036
1973	309	1.98	X_{29}	1.83	308.54	1.83	0.0025
1974	247	-20.06	X_{2}	-21.0	244.11	-21.0	0.0197
1975	270	9.31	X_{49}	9.25	269.85	9.25	0.0009
1976	324	20.00	X_{55}	19.0	321.30	19.00	0.0140
1977	337	4.01	X_{37}	4.15	337.45	4.15	0.0022
1978	355	5.34	X_{41}	5.31	354.89	5.31	0.0005
1979	322	-9.30	X_{9}	-9.25	322.16	-9.25	0.0008
1980	324	0.62	X_{25}	0.50	323.61	0.50	0.0020
1981	337	4.01	X_{37}	4.15	337.45	4.15	0.0022
1982	383	13.65	X_{51}	13.0	380.81	13.00	0.0096
1983	367	-4.18	X_{14}	-4.33	366.42	-4.33	0.0026
1984	403	9.81	X_{50}	9.75	402.78	9.75	0.0009

continue…

Year	Productivity	Percentage	Fuzzy set	FIM	Forecast	PP	FER
1985	473	17.37	X_{54}	17.50	473.53	17.50	0.0019
1986	453	-4.23	X_{14}	-4.33	452.52	-4.33	0.0018
1987	446	-1.55	X_{18}	-1.57	445.89	-1.57	0.0004
1988	397	-10.99	X_{7}	-11.0	396.94	-11.0	0.0003
1989	407	2.52	X_{31}	2.44	406.69	2.44	0.0013
1990	425	4.42	X_{38}	4.44	425.07	5.60	0.0024
1991	453	6.59	X_{44}	6.34	451.95	4.44	0.0003
1992	464	2.43	X_{31}	2.44	464.05	6.34	0.0039
1993	401	-13.58	X_{4}	-13.67	400.57	2.44	0.0002
1994	492	22.69	X_{57}	23.01	493.27	-13.7	0.0018
1995	404	-17.89	X_{3}	-17.0	408.36	23.01	0.0043
1996	400	-0.99	X_{20}	-0.99	400.00	-17.0	0.0178
1997	413	3.25	X_{34}	3.31	413.24	-0.99	0.0000
1998	478	15.74	X_{52}	15.0	474.95	3.31	0.0010
1999	420	-12.13	X_{6}	-12.33	419.06	15.00	0.0107
2000	454	8.10	X_{47}	8.25	454.65	-12.3	0.0037
2001	395	-13.00	X_{5}	-13.0	394.98	8.25	0.0024
2002	356	-9.87	X_{8}	-9.75	356.49	-13.0	0.0001
2003	375	5.34	X_{41}	5.31	374.90	-9.75	0.0023
2004	367	-2.13	X_{16}	-2.50	365.63	5.31	0.0004
2005	337	-8.17	X_{11}	-8.25	336.72	-2.50	0.0062
2006	541	60.53	X_{59}	61.0	542.57	-8.25	0.0014
2007	303	-43.99	X_{1}	-43.0	308.37	61.00	0.0048
2008	312	2.97	X_{33}	3.02	312.15	-43.0	0.0290
2009	381	22.12	X_{56}	22.34	381.70	3.02	0.0008

TABLE V COMPARISON OF DIFFERENT FORECASTING METHODS

Year	Productivity	Hao-Tien Liu	Enjian Bai et al.,	Proposed Method	Year	Productivity	Hao-Tien Liu	Enjian Bai et al.,	Proposed Method
1950	174	-	-	-	1980	324	330	330	323.61
1951	187	205	205	187.35	1981	337	398	370	337.45
1952	205	245	245	205.23	1982	383	365	365	380.81
1953	241	258	255	240.88	1983	367	370	370	366.42
1954	252	260	250	251.66	1984	403	448	425	402.78
1955	259	260	260	258.88	1985	473	435	455	473.53
1956	254	260	250	254.18	1986	453	435	445	452.52
1957	260	260	255	260.20	1987	446	395	395	445.89
1958	260	260	260	260.44	1988	397	392	392	396.94
1959	263	282	255	263.02	1989	407	448	425	406.69
1960	265	282	255	265.18	1990	425	455	455	425.07
1961	260	260	255	260.07	1991	453	435	465	451.95
1962	268	282	255	267.85	1992	464	405	405	464.05
1963	262	282	260	261.30	1993	401	448	425	400.57
1964	261	282	260	260.93	1994	492	405	405	493.27
1965	256	260	255	256.15	1995	404	448	425	408.36
1966	259	260	260	258.97	1996	400	392	405	400.00
1967	255	260	260	254.93	1997	413	465	455	413.24
1968	241	260	255	240.54	1998	478	435	455	474.95
1969	257	260	255	256.28	1999	420	465	455	419.06
1970	271	335	335	271.39	2000	454	435	465	454.65
1971	335	398	355	335.15	2001	395	392	405	394.98
1972	303	288	305	302.34	2002	356	350	375	356.49
1973	309	288	305	308.54	2003	375	365	365	374.90
1974	247	260	255	244.11	2004	367	370	365	365.63
1975	270	282	260	269.85	2005	337	398	355	336.72
1976	324	330	330	321.30	2006	541	305	305	542.57
1977	337	398	370	337.45	2007	303	288	288	308.37
1978	355	350	325	354.89	2008	312	385	385	312.15
1979	322	330	330	322.16	2009	381	365	365	381.70

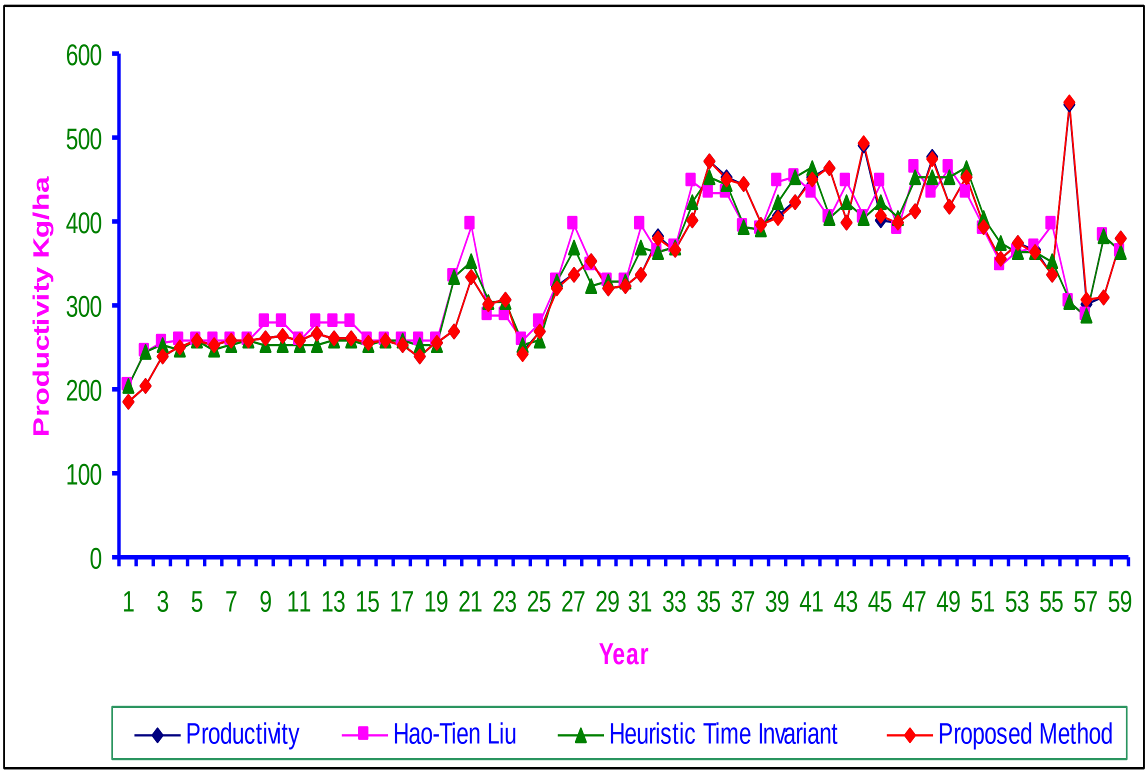

Fig. 2 Trends in Pulses productivity based on Hao-Tien Liu, Heuristic Time-Invariant fuzzy time-series and Proposed Method

TABLE VI CHARACTERISTICS OF MODEL PERFORMANCE PARAMETERS

	RMSE	MAE	MSE	AFER
Hao-Tien Liu (2007)	44.75	27.82	2003.03	0.1301%
Enjian Bai et al., (2011) (Heuristic time-invariant Fuzzy time series)	39.90	21.39	1592.27	0.1032%
Proposed Method	1.2446	0.6814	1.5489	0.0033%

